Terms & Forms of PFOL as Strings

(Exercise 43)

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EBNF

Extended Backus-Naur Form (Extended BNF)

- ISO/IEC 14977:1996(E)
- `http://www.iso.ch/cate/d26153.html` (~$65)
- `http://www.cl.cam.ac.uk/~mgk25/iso-14977.pdf`
- Final version of draft that project editor made available

Key Difference to BNF: Repeated Sequence

Example:

```
Fortran 77 continuation line = 5 * " ",
(character - (" " | "0")), 66 * [character] ;
```

SYNTAX OF PFOL: TERMS AND FORMULAS

T1 $\frac{x \in \mathcal{V}}{\textbf{term}_L[x]}$ T2 $\frac{c \in \mathcal{C}}{\textbf{term}_L[c]}$

T3 $\frac{x \in \mathcal{V}, \textbf{form}_L[\varphi]}{\textbf{term}_L[(\mathrm{I}\, x.\varphi)]}$

T4 $\frac{f \in \mathcal{F}(n\text{-ary}), \textbf{term}_L[t_1], \ldots, \textbf{term}_L[t_n]}{\textbf{term}_L[f(t_1, \ldots, t_n)]}$

F1 $\frac{p \in \mathcal{P}(n\text{-ary}), \textbf{term}_L[t_1], \ldots, \textbf{term}_L[t_n]}{\textbf{form}_L[p(t_1, \ldots, t_n)]}$

F2 $\frac{\textbf{form}_L[\varphi]}{\textbf{form}_L[\neg\varphi]}$ F3 $\frac{\textbf{form}_L[\varphi], \textbf{form}_L[\psi]}{\textbf{form}_L[(\varphi \Rightarrow \psi)]}$

F4 $\frac{x \in \mathcal{V}, \textbf{form}_L[\varphi]}{\textbf{form}_L[(\forall x.\varphi)]}$

EBNF FORM OF PFOL SYNTAX

$$
\begin{array}{llr}
\textbf{term}_L = & x \in \mathcal{V} & T1 \\
| & c \in \mathcal{C} & T2 \\
| & "(", "I", x \in \mathcal{V}, ".", \textbf{form}_L, ")" & T3 \\
| & n\text{-ary\_function}, "(", \textbf{term}_L, (n-1) * (",", \textbf{term}_L), ")" & T4
\end{array}
$$

$$
\begin{array}{llr}
\textbf{form}_L = & n\text{-ary\_predicate}, "(", \textbf{term}_L, (n-1) * (",", \textbf{term}_L), ")" & F1 \\
| & "\neg", \textbf{form}_L & F2 \\
| & "(", \textbf{form}_L, " \Rightarrow ", \textbf{form}_L, ")" | & F3 \\
| & "(", "\forall", x \in \mathcal{V}, ".", \textbf{form}_L, ")" & F4
\end{array}
$$

A Simple Example

The Equation

$(P(x) \Rightarrow \neg Q(x))$

With the following language: $L = (\mathcal{V}, \mathcal{C}, \mathcal{F}, \mathcal{P})$

$\mathcal{V} = \{x\}$, $\mathcal{C} = \{\}$, $\mathcal{F} = \{\}$, $\mathcal{P} = \{P, Q\}$

The Confirmation of the Syntax

$\textbf{term}_L[1] = x$	By T1
$\textbf{form}_L[1] = P(x)$	By F1
$\textbf{form}_L[2] = Q(x)$	By F1
$\textbf{form}_L[3] = \neg\,\textbf{form}_L[2]$	By F2
$\textbf{form}_L[4] = \textbf{form}_L[1] \Rightarrow \textbf{form}_L[3]$	By F3

A MORE COMPLICATED EXAMPLE

First, Expand:

$|x| \simeq$ if$(x < 0, -x, x)$

With the following language: $L = (\mathcal{V}, \mathcal{C}, \mathcal{F}, \mathcal{P})$

$\mathcal{V} = \{x, y, z\}$, $\mathcal{C} = \{0\}$, $\mathcal{F} = \{\text{abs}, -\}$, $\mathcal{P} = \{<, =\}$

Becomes (with a bit less shorthand):

$\varphi = \text{abs}(x) =$ **term**$_L$ by T4 and T1

$\theta =< (x, 0) =$ **form**$_L$ by F1, T1, and T2

$\psi =$ if$(\theta, -x, x)$

$= (Iy.\neg((\theta \Rightarrow = (y, -(x))) \Rightarrow \neg(\neg\theta \Rightarrow = (y, x)))) =$ **term**$_L$

$\varphi \simeq \psi$

$(\varphi \downarrow \vee \psi \downarrow) \Rightarrow (\varphi = \psi)$

$(\neg\exists z. = (z, \varphi) \Rightarrow \exists z. = (z, \psi)) \Rightarrow (= (\varphi, \psi))$

$((\forall z.\neg = (z, \varphi)) \Rightarrow \neg(\forall z.\neg = (z, \psi))) \Rightarrow (= (\varphi, \psi))$

Application of Syntax Rules: $\checkmark$

Syntactically Correct Formula!

CONCLUSION

- ➜ It's possible to define the set of terms and the set of formulas of PFOL as two sets of strings by mutual recursion.
- ➜ Using this method, we can verify the syntax of a PFOL expression.