

Topic 13 – Unlimited Register Machines (URMs)

Exercise 20 a) and b)

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Overview

- a) Define what a URM is.
 - ◆ Defining a URM
 - ◆ URM Instructions
 - ◆ A URM Program
- b) Construct a URM that computes the function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that $f(x) = 2 * x$.
 - ◆ Program
 - ◆ Example

a) Defining a URM

- A URM is an abstract model of a computer.
- It has a countably infinite sequence of registers labeled $R_0, R_1, R_2, \dots$. At any time the register R_n could store a natural number, r_n .

R_0	R_1	R_2	R_3	R_4	R_5	R_6	$\dots$
r_0	r_1	r_2	r_3	r_4	r_5	r_6	

a) URM instructions

Name	Notation	Effect
Zero	$Z(n)$	$r_n := 0$, and $r_0 := r_0 + 1$
Successor	$S(n)$	$r_n := r_n + 1$, and $r_0 := r_0 + 1$
Transfer	$T(m,n)$	$r_n := r_m$, and $r_0 := r_0 + 1$
Jump	$J(m,n,p)$	If $r_m = r_n$ then $r_0 := p$ else $r_0 := r_0 + 1$

a) URM Program

- A program consists of a finite sequence of these instructions $I_0, I_1, \dots, I_k$.

Input Convention

- Input $(x_0, \dots, x_n)$ by starting with $x_0, \dots, x_n$ in registers $R_0, \dots, R_n$ respectively and zero in the other registers.

Output Convention

- If a computation halts, the output is the number in R_1 , otherwise there is no output.
- The computation is denoted by $P(x)$.

b) Construct a URM

- The configuration of the URM that begins is $(1, x, x, 0, 0, \dots)$. Our program will add to R_1 x times using k as a counter:

R_0	R_1	R_2	R_3	R_4	...
I_n	$2 \cdot x$	x	k	0	

- The calculation will finish when $k = x$, leaving $2 \cdot x$ in R_1 .

b) URM $f(x) = 2 \cdot x$ Program

I_1 $J(3, 2, 5)$

I_2 $S(1)$

I_3 $S(3)$

I_4 $J(1, 1, 1)$

R_0	R_1	R_2	R_3	R_4	...
I_n	$2 \cdot x$	x	k	0	

b) Example – Compute $f(2) = 4$

Instruction	R_0	R_1	R_2	R_3	...
-	1	2	2	0	...
I_1 J(3,2,5)	2	2	2	0	
I_2 S(1)	3	3	2	0	
I_3 S(3)	4	3	2	1	
I_4 J(1,1,1)	1	3	2	1	
I_1 J(3,2,5)	2	3	2	1	
I_2 S(1)	3	4	2	1	
I_3 S(3)	4	4	2	2	
I_4 J(1,1,1)	1	4	2	2	
I_1 J(3,2,5)	5	4	2	2	

References

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