Base Negative Two

Presenter: Keith Mackay
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Bases

- A Base is the building block of mathematics.
- Most commonly used is base 10
- Other common bases include Bases 2, 8 and 16.

Historical Facts

- The Unary (Base 1) system has been dated to 37 000 years ago.
- Base 5 and 10 systems were not far behind.
- A method for writing numbers in base 10 is dated to approximately 9000 years ago.

Counting

Base 2	Base 8	Base 10	Base 16	Base -2
00000	00	00	00	00000
00001	01	01	01	00001
00010	02	02	02	00110
00011	03	03	03	00111
00100	04	04	04	00100
00101	05	05	05	00101
00110	06	06	06	11010
00111	07	07	07	11011
01000	10	08	08	11000
01001	11	09	09	11001
01010	12	10	0A	11110
01011	13	11	0B	11111
01100	14	12	0C	11100
01101	15	13	0D	11101
01110	16	14	0E	10011
01111	17	15	0F	10011
10000	20	16	10	10000

How to think of bases

Humans work almost entirely in base 10. So for simplicity all formulas will be provided in terms of base 10.

If we are given some x, which is separated into columns corresponding with its base, we can convert it to a base 10 numbers as follows:

$$x_n \times b^n + x_{n-1} \times b^{n-1} + \ldots + x_2 \times b^2 + x_1 \times b^1 + x_0 \times b^0$$

Example of conversion

Consider the following two examples:

$$01111_2 = 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$
$$= 0_{10} + 8_{10} + 4_{10} + 2_{10} + 1_{10}$$
$$= 15_{10}$$

$$13_8 = 1 \times 8^1 + 3 \times 8^0$$
$$= 8_{10} + 3_{10}$$
$$= 11_{10}$$

Example of Base –2

Base 2 example again:

$$01111_2 = 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$
$$= 0_{10} + 8_{10} + 4_{10} + 2_{10} + 1_{10}$$
$$= 15_{10}$$

Base (-2) example:

$$01111_{-2} = 0 \times (-2)^4 + 1 \times (-2)^3 + 1 \times (-2)^2 + 1 \times (-2)^1 + 1 \times (-2)^0$$
$$= 0_{10} + (-8)_{10} + 4_{10} + (-2)_{10} + 1_{10}$$
$$= -5_{10}$$

Properties of Base -2

- A negative number has an even number of bits, A positive number has an odd number of bits.

- Negation looks at the LSB, if it is a 0, it recursively looks at LSB + 1. If it is a 1, then for every odd bit (1, 3, 5…), the bit to the left is inverted, except for the MSB.

Properties of Base -2

- Shifting the bits to the left is a multiplication by -2, and shifting to the right a division by -2.
- Increment is fairly straight forward, consider increment in binary, but remember that base -2 relies on successive bits being the negative of the previous.
  (i.e. carry 11 as opposed to simply 1)
- Addition and subtraction are similarly based on increment.

Practical examples

- In the 1950s, Poland produced the only two computers based on Base -2, called SKRZAT 1, and BINEG. Unfortunately, they never caught on.

- Data transfers can be smaller as we are no longer restricted to transfer data in terms of 32- or 64-bit words.

Problems with Base -2

- Very difficult to understand, it is not very intuitive.
- The idea of negative bases as only been around since 1885, and was not even looked at again until 1936.

References

- Base Valued Numbers
  http://www.psinvention.com/zoetic/basenumb.htm
- Mathworld: Base
  http://mathworld.wolfram.com/Base.html
- The art of Computer Programming
  Volume 2: Seminumerical Algorithms, Donald E. Knuth