

Hilbert Choice Operator & Quantifications

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Definition of Hilbert Choice Operator ε

- $\varepsilon_x P(x)$ denotes
 - an object c , such that $P(c)$ holds,
 - given there is such an object
 - an arbitrary object,
 - otherwise



$$\forall x P(x) \equiv P(\varepsilon_x (\neg P(x)))$$

$$\varepsilon_x (\neg P(x)) \quad P(\varepsilon_x (\neg P(x))) \quad \forall x P(x)$$



a

F

F



c

F

F



d

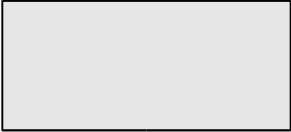
T

T

x in yellow : $P(x) = F$
 x in green: $P(x) = T$



$$\exists x P(x) \equiv P(\varepsilon_x P(x))$$

	$\varepsilon_x P(x)$	$P(\varepsilon_x P(x))$	$\exists x P(x)$
	a	F	F
	b	T	T
	d	T	T

x in yellow : $P(x) = F$

x in green: $P(x) = T$



More about ε

- Implementation in Theorem Proving Systems
 - ε is implemented in HOL, Isabelle and PVS
Introduce of ε implies introduce of Axiom of choice
 - Iota (definite description) is implemented in IMPS
Iota is only for convenience purpose, can be eliminated



References

- <http://www.hf.uio.no/filosofi/njpl/vol1no2/howlogic/>
- <http://www.cs.nott.ac.uk/~gmh/hol-review.ps>
- <http://www4.informatik.tu-muenchen.de/papers/MS97.ps.gz>

