

Dense Linear Orders, with FOL

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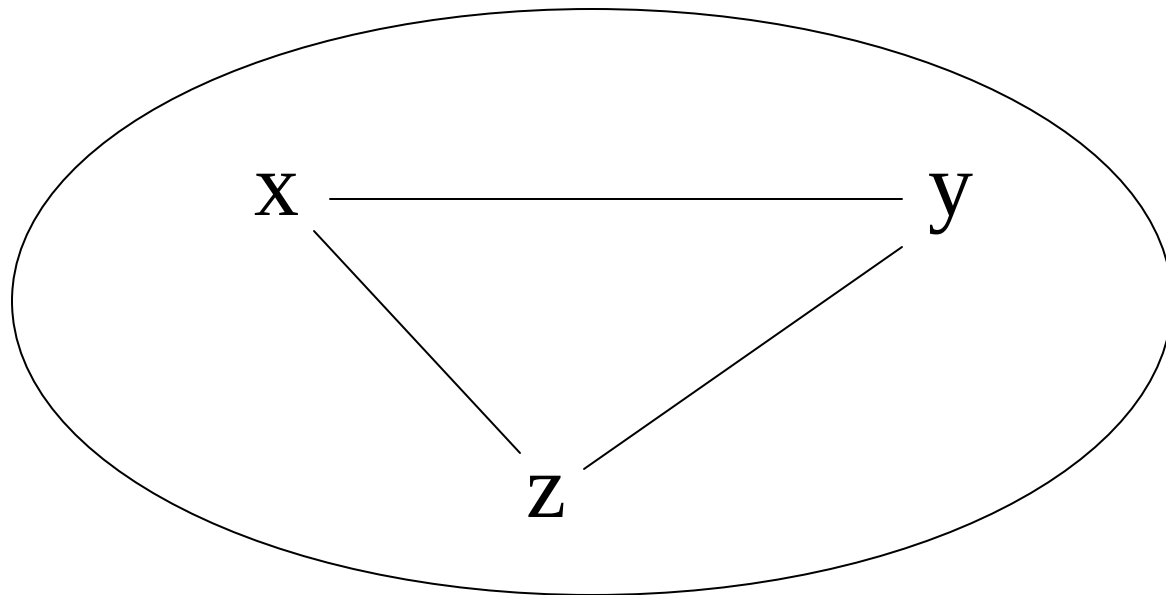
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What is Dense Linear Order?

- Reflexivity
- Antisymmetry
- Transitivity
- Comparability

- *Density*
- *Infinity*

Density



Formalization in FOL

$$T = (L, \Gamma)$$

$$L = (V, C, F, P)$$

- V: Symbols
- C: Constants
- F: Functions
- P: Predicates

Γ : Axioms

Formalization of Dense Linear Orders

$$L = (V, \Phi, \Phi, \{\leq\})$$

Γ : Axioms

1. $\forall x. (x \leq x)$ (*Reflexivity*)
2. $\forall x, y. (x \leq y) \wedge (y \leq x) \Rightarrow (x = y)$ (*Antisymmetry*)
3. $\forall x, y, z. (x \leq y) \wedge (y \leq z) \Rightarrow (x \leq z)$ (*Transitivity*)
4. $\forall x, y. (x \leq y) \vee (y \leq x)$ (*Comparability*)
5. $\forall x, y, \exists z. (x \leq y) \wedge \neg(x = y) \Rightarrow$
 $(x \leq z) \wedge (z \leq y) \wedge \neg(z = x) \wedge \neg(z = y)$ (***Density***)
6. $\exists x, y. \neg(x = y)$ (***Infinity***)

Models

1. $(\mathbb{R}, \leq)$
2. $(\mathbb{Q}, \leq)$
3. $(\{q \mid q \in \mathbb{Q} \wedge q \in [0,1]\}, \leq)$ (DLO with *end points*)

References

1. D. Pincus, “*The Dense Linear Ordering Principle*”, *The Journal of Symbolic Logic*, 62(2):438-456, June 1997.
2. E. Mendelson, *Introduction to Mathematical Logic*, 3rd Ed.