Nonstrict Linear Order vs. Strict Linear Order

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Properties Define Nonstrict Linear Order ($\preceq$)

- Reflexivity. $\forall\, x \in A,\, x \preceq x$.
- Antisymmetry. $(x \preceq y) \wedge (y \preceq x)$ implies $(x = y)$.
- Transitivity. $(x \preceq y) \wedge (y \preceq z)$ implies $(x \preceq z)$.
- Comparability. $\forall\, x,\, y \in A,\, (x \preceq y) \vee (y \preceq x)$.

Properties Define Strict Linear Order ($\prec$)

- Irreflexivity. $\forall\, x \in A, \neg\, (x \prec x)$.
- Transitivity. $(x \prec y) \wedge (y \prec z)$ implies $(x \prec z)$.
- Trichotomy. $\forall\, x,\, y \in A, (x \prec y) \vee (y \prec x) \vee (x = y)$.

$(A, \preceq) \Longrightarrow (A, \prec)$

- Irreflexivity

$$\begin{array}{ll}
 & \neg\ (x \prec x) \\
\Longleftrightarrow & \neg\ ((x \preceq x) \wedge \neg\ (x = x)) \qquad \text{(def. of } \prec) \\
\Longleftrightarrow & \neg\ (x \preceq x) \vee (x = x) \\
\Longrightarrow & \text{true}
\end{array}$$

- Transitivity

$$\begin{array}{lll}
 & (x \prec y) \wedge (y \prec z) \text{ implies } (x \prec z) & \\
\Longleftrightarrow & ((x \preceq y) \wedge \neg(x = y)) \wedge ((y \preceq z) \wedge \neg(y = z)) & \\
 & \text{implies } ((x \preceq z) \wedge \neg(x = z)) & \text{(def. of } \prec) \\
\Longleftrightarrow & ((x \preceq y) \wedge (y \preceq z) \wedge \neg(x = y) \wedge \neg(y = z)) & \\
 & \text{implies } ((x \preceq z) \wedge \neg(x = z)) & \\
\Longleftrightarrow & ((x \preceq z) \wedge \neg(x = z)) & \\
 & \text{implies } ((x \preceq z) \wedge \neg(x = z)) & \text{(transitivity of } \preceq\ ) \\
\Longrightarrow & \text{true} &
\end{array}$$

- Trichotomy

$$
\begin{array}{lll}
 & x \prec y \vee y \prec x \vee x = y & \\
\iff & ((x \preceq y) \wedge \neg(x = y)) \vee ((y \preceq x) & \\
 & \wedge \neg\ (y = x)) \vee x = y & \text{(def. of } \prec\text{)} \\
\iff & \neg(x = y) \wedge ((x \preceq y) \vee (y \preceq x)) & \\
 & \vee\ x = y & \\
\iff & \neg(x = y) \wedge \text{true} \vee x = y & \text{(comparability of } \preceq\text{)} \\
\iff & \neg(x = y) \vee x = y & \\
\Longrightarrow & \text{true} &
\end{array}
$$

Similarly, we can prove $(A, \prec) \Longrightarrow (A, \preceq)$.

So $(A, \prec) \iff (A, \preceq)$.

Properties

A binary relation R in set A:

Reflexivity	$\forall\, x \in A,\ (x, x) \in R$.
Irreflexivity	$\forall\, x \in A,\ \neg\ (x, x) \in R$.
Symmetry	$(x, y) \in R$ implies $(y, x) \in R$.
Asymmetry	$(x, y) \in R$ implies $\neg\ (y, x) \in R$.
Antisymmetry	$(x, y) \in R \wedge (y, x) \in R$ implies $x = y$.
Transitivity	$(x, y) \in R \wedge (y, z) \in R$ implies $(x, z) \in R$.