CAS 701

Logic and Discrete Mathematics in Software Engineering

EXERCISE 41B

Let f: N-> N generate the Fibonacci sequence. Define f by recursion via a monotone functional.

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Fibonacci Sequence

- The Fibonacci numbers are defined by the following recurrence:

  $F_0 = 0$

  $F_1 = 1$

  $F_i = F_{i-1} + F_{i-2}, i \geq 2$

- The sequence:

  0,1,1,2,3,5,8,13,21,34,55,..

Recursion via a monotone functional

R = (A,fib,F)   where

- A=(L,□): standard theory of real Arithmetic

- fib is a constant of type **N** **N** not in L

- F = □g:**N** **N**. □ n:**N**. if (n<2, n, g(n-1)+g(n-2))

F is a monotone functional

- fib = F fib, observe:

$F^0 \varnothing = \varnothing$ = { }

$F^1 \varnothing = F \varnothing$ = { 0 -> 0 }

$F^2 \varnothing = F (F \varnothing)$ = { 0 -> 0, 1-> 1 }

$F^3 \varnothing = F (F (F \varnothing))$ = { 0-> 1, 1 -> 1, 2 -> 1 }

$F^4 \varnothing = F (F (F (F \varnothing)))$ = { 0-> 1, 1 -> 1, 2 -> 1, 3 -> 2 }

$\varnothing \subseteq F \varnothing \subseteq F^1 \varnothing \subseteq F^2 \varnothing \subseteq F^3 \varnothing \subseteq \ldots \subseteq F^{1,000,000} \varnothing \subseteq \ldots$

- $F^{\omega} \varnothing$ is the least fixed point of F

Recursion via a monotone functional

- The defining axiom of R is □ which says “fib is a least fixed point of F”.
- The definitional extension resulting from R is the theory (L U {fib}, □ U {□}).

References

- W. M. Farmer. A Scheme for Defining Partial Higher-Order Functions by Recursion. McMaster University, Ontario, 2001.
- S. Abramsky, Dov M. Gabbay, and T.S.E. Maibaum. Handbook of logic in computer science, v. 1. Oxford : Clarendon Press, 1992.