Presentation

Exe 15

Suppose P=(S,<=) is preorder. Define a nontrivial equivalence relation R on S such that the quotient structure P/R is partial order

By Zhuomei

Preorder

A binary relation <= over a set S is a preorder if it has following properties:

- Reflexive: for all a in S, a <= a
- Transitive: for all a,b and c in S,
  if a <= b and b <= c then a <= c

Partial Order

A binary relation <= over a set S is a partial order if it has following properties:

- Reflexive: for all a in S, a <= a
- Transitive: for all a,b and c in S,
  if a <= b and b <= c then a <= c
- Antisymmetric:
  for all a, b in S, if a ~ b and b ~a
  then a = b

Equivalence Relation

A binary relation **~** over a set S is a equivalence relation if it is

- Reflexive: for all a in S, a **~** a
- Transitive: for all a,b and c in S,
  if a ~ b and b ~ c then a ~ c
- Symmetric: for all a, b in S,
  if a ~ b then b ~a

Quotient Set

Given a set S and an equivalence relation ~ over S, the quotient set written as S/~, is the set of all equivalence classes in S under equivalence relation ~.

Example:

If S is the set of all cars and ~ is the equivalence relation of " having the same color", then the set of all green cars and the set of all white cars are different equivalence classes. S/~ could be identified with the set of all car colors.

Equivalence Class

Given a set S. a is an element in set S. Equivalence class is a subset of S, and satisfies {x in S| x ~ a}where ~ is an equivalence relation. This equivalence class is denoted as [a]

- Any two equivalence classes are either equal or disjoint, that is, the set of all equivalence classes of S forms a partition of S
- The property of an equivalence relation a ~ b if and only if [a] = [b]

Suppose P = (S, <=) is preorder. Define a nontrivial equivalent relation R on S such that the quotient structure P/R is partial order

We may take two steps:

- Find an equivalence relation R over set S
- Find a binary relation ~ over the set of equivalence classes and then prove this binary relation ~ is a partial order

we define a binary relation R on set S: for all a,b in S,
if a <= b and b <= a then a R b (<= is preorder on S)

Prove binary relation R is an equivalence relation

- for all a in S, a <= a → a R a   (reflexive)
- for all a, b and c in S, if a R b and b R c, then

  a <= b, b<=c and c <= b, b<=c

  → a <= c and c <= a

  → a R c   (transitive)
- for all a, b in S, from a R b, we can get

  a <= b, b <= a

  → b R a   (symmetric)

We can get conclusion that R is an equivalence relation over set S.

Define a binary relation ~ on the set of equivalence classes:
for all elements of Set S a and b, if a <= b, define [a] ~ [b]

Prove that binary relation ~ is a partial order

- reflexive a <= a → [a] ~ [a]
- transitive [a] ~ [b] and [b] ~ [c]

  → a <= b and b <= c

  → a <= c (<= has transitive property)

  → [a] ~ [c]
- antisymmetric

  [a] ~ [b] and [b] ~ [a]

  → a <= b and b <= a

  → a R b (definition of equivalence relation R)

  → [a] = [b] (property of equivalence relation)

That means that ~ is a partial order on the set of equivalence classes