Theory for Stacks in BESTT

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Content will include:

- Theory of Stacks of Integers
- Theory of Stacks of abstract elements
- Theory of Stacks of lists of abstract elements
- conclusion

Stacks of Integers T_1

- Theory $\mathbf{T_1} = (L, \Gamma)$
- $L=(K, V, C, \tau)$
- $K=(A ,B)$
- $C = \{0, S, \varepsilon, push, pop, top, IsEmpty\} \cup C_0$
- $B=\{\mathbb{N}\} \cup B_0$

Type of the Constants of T_1

- $\tau(0)=\mathbb{N}$
- $\tau(S)=(\mathbb{N}\rightarrow\mathbb{N})$
- $\tau(\varepsilon)=Stack[\mathbb{N}]$
- $\tau(push)=((Stack[\mathbb{N}]\ X\ \mathbb{N})\rightarrow Stack[\mathbb{N}])$
- $\tau(pop)=(Stack[\mathbb{N}]\rightarrow Stack[\mathbb{N}])$
- $\tau(top)=(Stack[\mathbb{N}]\rightarrow\mathbb{N})$
- $\tau(IsEmpty)=(Stack[\mathbb{N}]\rightarrow *)$

Axioms of Stacks in T_1

- IsEmpty(ε)=T
- $\forall$ x:ℕ and $\forall$ s: Stack[ℕ]
- IsEmpty(push (s,x))=F
- top(push(s,x))=x
- pop(push(s,x))=s
- top(ε) = $\perp_{ℕ}$
- pop(ε) = $\perp_{ℕ}$
- $\forall$ P: (Stack[ℕ]→*){[P(ε) Λ ($\forall$ x: N $\forall$ s:Stack[N](P(s)=>P(push(s, x)))] => [$\forall$ s:stack[ℕ]. P(s)]}

Axioms of Natural Numbers of T_1

- $\forall\ x: \mathbb{N}\ S(x)=0$
- $\forall\ x,y: \mathbb{N}\ S(x)=S(y) \rightarrow x = y$
- $\forall\ P: (\mathbb{N}\rightarrow *)\{[P(0) \wedge (\forall\ x: N\ (P(x) \Rightarrow P(S(x))))] \Rightarrow [\forall\ x:\mathbb{N}.\ P(x)]\}$

Stacks of Abstract Elements

- Theory $\mathbf{T_2} = (L, \Gamma)$
- $L=(K, V, C, \tau)$
- $K=(A, B)$
- $C = \{ \varepsilon, push, pop, top, IsEmpty\} \cup C_1 \cup C_0$
- $B=B_1 \cup B_0$

Type of the Constants of T_2

- let $\alpha \in B_1$
- $\tau(\varepsilon)=Stack[\alpha]$
- $\tau(push)=((Stack[\alpha] \times \alpha) \rightarrow Stack[\alpha])$
- $\tau(pop)=(Stack[\alpha] \rightarrow Stack[\alpha])$
- $\tau(top)=(Stack[\alpha] \rightarrow \alpha)$
- $\tau(IsEmpty)=(Stack[\alpha] \rightarrow *)$

Axioms of Stacks of T_3

- IsEmpty(ε)=T
- ∀ x:α and ∀s:Stack[α]
- IsEmpty(push(s,x))=F
- top(push(s,x))=x
- pop(push(s,x))=s
- top(ε) = ⊥α;   pop(ε) = ⊥α
- ∀ P: (Stack[α▌→*){[P(ε) Λ (∀ x: α ∀ s:Stack[α](P(s)=>P(push(s, x)))] => [∀ s:stack[α▌. P(s)]}
- All axioms of β

Stacks of Lists

- Theory $\mathbf{T_3} = (L, \Gamma)$
- $L=(K, V, C, \tau)$
- $K=(A, B)$
- $C = \{\varepsilon, push, pop, top, IsEmpty\} \cup C_1 \cup C_0$
- $B=B_1 \cup B_0$

Type of the Constants of T_3

- let $\beta \in B_1$; $\alpha = lists[\beta]$
- $\tau(\varepsilon)=Stack[\alpha]$
- $\tau(push)=((Stack[\alpha] \times \alpha)\rightarrow Stack[\alpha])$
- $\tau(pop)=(Stack[\alpha] \rightarrow Stack[\alpha])$
- $\tau(top)=(Stack[\alpha] \rightarrow \alpha)$
- $\tau(IsEmpty)=(Stack[\alpha]\rightarrow *)$

Axioms of Stacks of T_3

- IsEmpty(ε)=T
- ∀ x:α and ∀s:Stack[α]
- IsEmpty(push(s,x))=F
- top(push(s,x))=x
- pop(push(s,x))=s
- top(ε) = ⊥α;   pop(ε) = ⊥α
- ∀ P: (Stack[α]→*){[P(ε) Λ (∀ x: α ∀ s:Stack[α](P(s)=>P(push(s, x)))] => [∀ s:stack[α]. P(s)]}
- All axioms of β

Axioms of Lists of T_3

- $\forall$ z:α and $\forall$l:lists[α]
- hd(cons(z,l))=z
- tl(cons(z,l))=l
- hd(nil) = $\perp_\alpha$
- tl(nil) = nil

Conclusion

- Stack Theory can easily expressed as a general form
- Do not need to express partial functions explicitly
- Built in types & functions help to keep the expressions succinct and straightforward

References

- W. M. Farmer, "A basic extended simple type theory", Technical Report, 12 pp., McMaster University, 2001.
- W. M. Farmer, "A partial functions version of Church's simple theory of types", Journal of Symbolic Logic, 55:1269-1291,1990.
- http://www.cis.ohio-state.edu/rsrg/documents/ Harms /chapter2.pdf

Questions?

Comments

- Hi, friends:
- In order to save my computer, I've get rid of the blue grids.
- According to Dr. Farmer's instruction, I've increased one axiom to each stack, which help to build the effective proof and make sure there will be no two empty stack of the same type.
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