Topic 12: Show how to construct a Universal Turing Machine

Prepared and Presented by: Joe Fakhri

Revised October 16, 2002

Outline

- What is a Turing Machine? (a quick intro)

- Construction of a Universal Turing Machine

What is a Turing Machine?

- Formal definition: it is a quadruple $M=(K,\Sigma,\delta,s)$
- K is a finite set of states
- $s \in K$ is the initial state
- Σ is a finite set of symbols (Σ is the alphabet)
- Σ always contains the special symbols $\sqcap$ and ▶ denoting the blank and first symbols

What is a Turing Machine? (continued)

- δ is the transition function which maps:

$K \times \Sigma$ to $(K \cup \{h, \text{“yes”}, \text{“no”}\}) \times \{\leftarrow, \rightarrow, -\}$

- Function δ is the “program” of the machine. For each current state $q \in K$ and a current symbol $\sigma \in \Sigma$, it specifies a triple $\delta(q, \sigma) = (p, \rho, D)$, where p is the next state, ρ is the symbol to be overwritten on σ, and $D \in \{\leftarrow, \rightarrow, -\}$
- The input string is initialized to a ► followed by a finitely long string $x \in (\Sigma - \sqcap)$

Construction of a Universal Turing Machine

- A universal Turing machine interprets the input as a description of another Turing machine M, concatenated with a description of the input x.

- We denote it as U(M; x) = M(x)

- Assumptions:
  - no priori bounds on number of states and symbols that U must face

  $\therefore$ both states and their symbols are integers

Construction of a Universal Turing Machine (continued)

- For any Turing machine $M=(K,\Sigma,\delta,s)$ we assume :
  - $\Sigma = \{1,2, \ldots , |\Sigma|\}$ and $K = \{|\Sigma| + 1, |\Sigma| + 2, \ldots , |\Sigma| + |K|\}$
  - $|\Sigma| + 1$ is the starting state
  - Numbers $|K| + |\Sigma| + 1, \ldots , |K| + |\Sigma| + 6$ will encode the special symbols.
  - All numbers will be processed by U as binary numbers with $\lceil (\log|K|+|\Sigma| + 6) \rceil$ bits
  - Introduce leading zeros to make all numbers of the same length

Construction of a Universal Turing Machine (continued)

- A description of the Turing machine M will start by the number |K| in binary, followed by |Σ|, followed by a description of δ as $((q, \sigma), (p, \rho, D))$.
  - Symbols "(", ")", ",", "," etc $\in$ Alphabet of U

- The description of M is followed by a ";" followed by a description of the input x.
  - x's symbols are also encoded in binary integers separated by ","

Construction of a Universal Turing Machine (continued)

- The universal Turing machine U on input M; x simulates M on input x (it seems U has two strings)
- U uses its second string to store M's configuration
  - Configuration of the following form (w, q, u)
- To simulate a step of U:
  - U scans its second string until it finds the binary description of an integer corresponding to a state
  - It searches the first string for a rule of δ matching the current state
  - If a rule is located, M moves to the left in the 2^{nd} string to compare symbols
  - If no match then another rule is sought
  - If there is a match the rule is implemented (ie. Change current symbol, state and direction)

References

- "Computational Complexity", by Christos H. Papadimitriou
- Introduction to Automata Theory, Languages, and Computation, by John E. Hopcroft, Rajeev Motwani, and Jeffrey D. Ullman, SECOND EDITION
- http://mathworld.wolfram.com/TuringMachine.html
- Note: This presentation is posted on : www.cas.mcmaster.ca/~fakhrijm/home.html