The Fibonacci Sequence

An Example of well-founded Recursion

By

Dawn MacIsaac

MEng Candidate

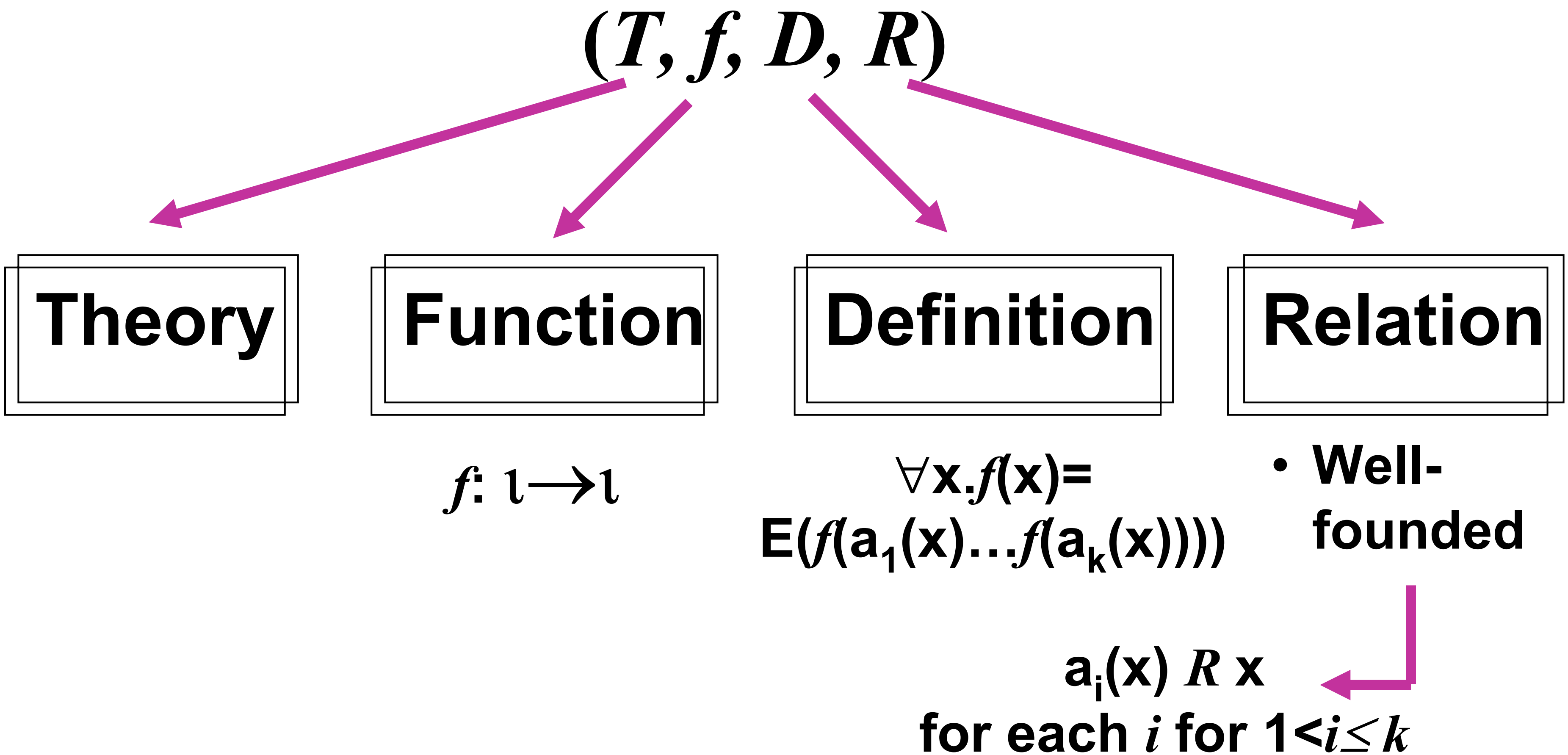
Department of Computing and Software

“How many pairs of rabbits will be produced in a year, beginning with a single pair, if every month each pair bares a new pair which become productive from the second month on?”

Month	Infants	Adults	Total
0	1	0	1
1	0	1	1
2	1	1	2
3	1	2	3
4	2	3	5
5	3	5	8
6	5	8	13
⋮	⋮	⋮	⋮

Month	Infants	Adults	Total
0	1	0	1
1	0	1	1
2	1	1	2
3	1	2	3
4	2	3	5
5	3	5	8
6	5	8	13
⋮	⋮	⋮	⋮

Well-Founded Recursion



Well-Founded Recursion

$$(T, f, D, R)$$

Theory

- Peano's Arithmetic (PA)

Well-Founded Recursion

$$(T, f, D, R)$$

Function

$$f\text{: } \iota \rightarrow \iota$$

Well-Founded Recursion

$$(T, f, D, R)$$

Definition

$$f(n) = \text{if}(n=0,\ 1,\ \text{if}(n=1,\ 1,\ f(n-1) + f(n-2)))$$

Well-Founded Recursion

(T, f, D, R)

Relation

[n-1<n] Λ [n-2<n]

(<)

Well-Founded Recursion

$$(T, f, D, R)$$

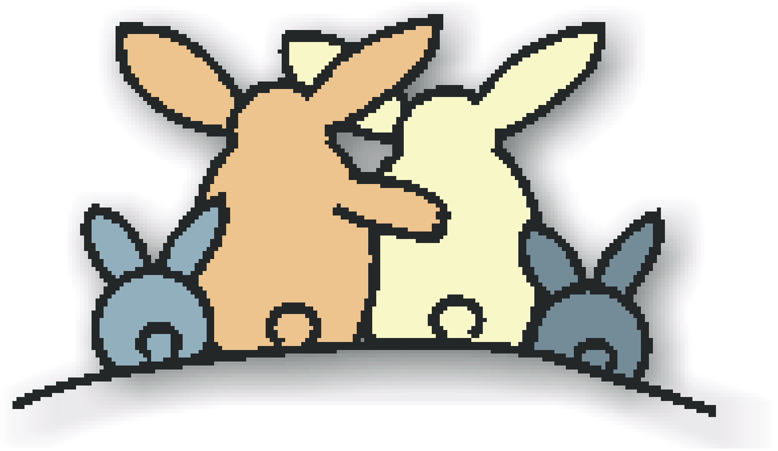
(PA, f: $\iota \rightarrow \iota$,

f(n) = if(n=0, 1, if(n=1, 1, f(n-1) + f(n-2))),

<)

Non-Recursive Definition

$$f(n) = \frac{1}{\sqrt{5}}\left[\left[\frac{1+\sqrt{5}}{2}\right]^{n} - \left[\frac{1-\sqrt{5}}{2}\right]^{n}\right]$$



Any Questions?