Computing and Software 701

Logic and Discrete Mathematics In Software Engineering

Fall 2004

Exercise Group 1

Due October 7, 2004

Revised: 23 September 2004

In the following exercises, let **H** be the Hilbert-style proof system for the propositional language L_0 presented in class.

1. [2 pts.] Exercise 4 on p. 66 of Grimaldi.
2. [2 pts.] a) and c) of exercise 6 on p. 66 of Grimaldi.
3. [2 pts.] Exercise 18 on p. 67 of Grimaldi.
4. [4 pts.] b), d), f), and h) of exercise 10 on p. 85 of Grimaldi.
5. [6 pts.] Exercise 12 on p. 86 of Grimaldi.
6. [2 pts.] Assume that $\{\neg, \Rightarrow\}$ is complete and then show that $\{|\}$ is complete.
7. [3 pts.] Using truth tables show that each logical axiom of **H** is a tautology.
8. [5 pts.] Define what it means for a formula of propositional logic to be in *disjunctive normal form* and in *conjunctive normal form*. Let L be a language of propositional logic with the connectives $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$. Write an algorithm that, given a formula A of L as input, returns a formula A' as output such that A' is in conjunctive normal form and $A \Leftrightarrow A'$ is a tautology.
9. [3 pts.] Prove in **H** that $\neg\neg A \Rightarrow A$ is a tautology.
10. [5 pts.] Let Σ be a set of formulas of L_0. Assuming that **H** is complete, prove that if Σ is consistent in **H** then Σ is satisfiable.