Computing and Software 701

Logic and Discrete Mathematics In Software Engineering

Fall 2004

Exercise Group 2

Due October 21, 2004

Revised: 5 October 2004

Justify your answers in each of the following exercises.

1. [4.5 pts.] Exercise 8 on p. 134 of Grimaldi.
2. [3 pts.] Exercise 2 on p. 146 of Grimaldi.
3. [6 pts.] Exercise 4 on p. 146 of Grimaldi.
4. [6 pts.] Exercise 6 on p. 146 of Grimaldi.
5. [4 pts.] State and prove the de Morgan Laws for sets.
6. [4 pts.] Exercise 6 on p. 258 of Grimaldi.
7. [2 pts.] Exercise 18 on p. 259 of Grimaldi.
8. [3 pts.] Exercise 2 on p. 265 of Grimaldi.
9. [3 pts.] Exercise 4 on p. 265 of Grimaldi.
10. [1 pt.] Exercise 4 on p. 288 of Grimaldi.
11. [6 pts.] Exercise 8 on p. 288 of Grimaldi.
12. [6 pts.] Let $f : A \to B$ and $g : B \to C$ be total, and let $h = g \circ f : A \to C$ be the composition of g and f.

    (a) Prove that, if f and g are injective, then h is injective, but the converse is false.

(b) Prove that, if f and g are surjective, then h is surjective, but the converse is false.

13. [3 pts.] What is the cardinality of the function space $\mathbf{N} \rightarrow \mathbf{N}$, where $\mathbf{N}$ denotes the set of natural numbers?

14. [8 pts.]

    (a) Let T_n be a full binary tree of height $n \geq 1$. What is the cardinality of the set of nodes in T_n? What is the cardinality of the set of paths in T_n?

    (b) Let T_∞ be a full binary tree of infinite height. What is the cardinality of the set of nodes in T_∞? What is the cardinality of the set of paths in T_∞?

15. [4 pts.] Exercise 4 on p. 343 of Grimaldi.

16. [9 pts.] Exercise 16 on p. 344 of Grimaldi.

17. [3 pts.] Show how a relation $R \subseteq A \times B$ can be transformed into an "equivalent" total function $f_R : A \rightarrow \mathcal{P}(B)$, where $\mathcal{P}(B)$ is the power set of B. (A function like f_R is sometimes called a *many-valued function.*)

18. [8 pts.]

    (a) Suppose $R \subseteq A^2$ is an equivalence relation. Let the *equivalence class* of $a \in A$ be the set $\{b \mid aRb\}$. Show that the set of equivalence classes is a *partition* of A.

    (b) Suppose P is a partition of A. Let $R \subseteq A^2$ be the relation such that aRb iff, for some $C \in P$, $a, b \in C$. Show that R is an equivalence relation.