Computing and Software 701

Logic and Discrete Mathematics In Software Engineering

Fall 2004

Exercise Group 4

Due November 25, 2004

Revised: November 11, 2004

Justify your answers in each of the following exercises.

1. [3 pts.] Suppose $P = (S, \leq)$ is a preorder. Define a nontrivial equivalence relation R on S such that the *quotient structure* P/R is a partial order.

2. [3 pts.] Define the *transitive closure* of a binary relation. Prove that the transitive closure of a union of equivalence relations is an equivalence relation.

3. [4 pts.] Prove that a term rewriting system is Church-Rosser iff it is confluent.

4. [6 pts.] Let $T = (L, \Gamma)$ be a theory of groups in FOL where

$$L = (\{e\}, \{\mathsf{mul}, \mathsf{inv}\}, \{=\})$$

with mul binary and inv unary and Γ is the set of the following formulas of L:

   (a) $\forall x, y, z \,.\, x \mathrel{\mathsf{mul}} (y \mathrel{\mathsf{mul}} z) = (x \mathrel{\mathsf{mul}} y) \mathrel{\mathsf{mul}} z$.

   (b) $\forall x \,.\, x \mathrel{\mathsf{mul}} e = x$.

   (c) $\forall x \,.\, e \mathrel{\mathsf{mul}} x = x$.

   (d) $\forall x \,.\, x \mathrel{\mathsf{mul}} \mathsf{inv}(x) = e$.

   (e) $\forall x \,.\, \mathsf{inv}(x) \mathrel{\mathsf{mul}} x = e$.

   Construct a term rewriting system that is sound and complete with respective to T, finite, confluent, and finitely terminating.

5. [3 pts.] Exercise 2 on p. 208 of Grimaldi.
6. [3 pts.] Exercise 12 on p. 219 of Grimaldi.
7. [3 pts.] Exercise 6 on p. 245 of Grimaldi.
8. [3 pts.] Prove the deduction theorem for propositional logic by induction on the structure of formulas
9. [3 pts.] Show that the exponential function on $\mathbf{N}$ is primitive recursive.
10. [3 pts.] Give a natural example of a well-founded relation that is not a partial order.
11. [3 pts.] Show that Ackermann's function is an instance of well-founded recursion.
12. [3 pts.] Define the set of terms and the set of formulas of a language L of FOL as two sets of strings by mutual recursion.
13. [6 pts.] Let $f : \mathbf{N} \rightarrow \mathbf{N}$ generate the Fibonacci sequence.
    (a) Show that f is a primitive recursive function.
    (b) Define f by well-founded recursion.
    (c) Define f by recursion via a monotone functional.
14. [3 pts.] Construct a monotone functional $F : \alpha \rightharpoonup \alpha$ such that the least fixed point of F is $F^{\gamma}(\triangle_{\alpha})$ where $\omega < \gamma$ (i.e., γ is an ordinal greater than ω) and $\triangle_{\alpha}$ is the empty function of sort α.