Computing and Software 701

Logic and Discrete Mathematics In Software Engineering

Fall 2004

Exercise Group 5

Due December 8, 2004

Revised: November 23, 2004

1. [10 pts.] Let $L = (\{R\}, \tau)$ be a language of STT where

$$\tau(R) = (\iota \to (\iota \to *)).$$

   Formalize a theory $T = (L, \Gamma)$ in simple type theory such that $M = (\mathcal{D}, I, e)$ is model of T iff $I(R)$ is a well-founded relation on D_ι.

2. Let a *tree* be defined by:

   - Every real number is a tree.
   - If s and t are trees, then the pair (s, t) is a tree.

   (a) [10 pts.] Formulate a theory of trees in many-sorted STT similar to Peano arithmetic.

   (b) [10 pts.] Define the "mirror" of a tree by well-founded recursion and prove by induction that the composition of the mirror function with itself is the identity function on trees.

3. [10 pts.] Formalize a theory of stacks of integers in many-sorted STT.