Church’s Lambda Calculus

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Introduction

In 1936, three formal approcahes to computability were proposed

- Turing machines
- Kleene's recursive function theory
- Church's Lambda Calculus.

Lambda calculus

The λ calculus is a formal system based on:

- function definition
- function application
- function recursion

λ expressions

- <expression> :: = <name> | <function> | <application>

- <function> :: = λ<name>. <body>
  <body> :: = <expressions>

- <application> :: = <function expression> <argument expression>
  where:
  <function expression> :: = <expression>
  <argument expression> :: = <expression>

Example:
The function "add-two" f(x) = x + 2 is expressed in λ calculus as

λ x. x + 2

Free and bound variables

- free (V):= V
- free (E.E'):= free(E)$\cup$ free (E')
- free (λ V.E):= free(E)$\setminus$V
- bound (V):= $\{\}$
- bound (E.E'):= bound(E)$\cup$ bound (E')
- bound (λ V.E):= bound(E)$\cup$ V

Substitution

- $\triangleright$ V[V/E]= E
- $\triangleright$ W[V/E]=W
- $\triangleright$ (λ V. F)[V/E]= λ V. F
- $\triangleright$ (λ W. F)[V/E]= λ W. (F[V/E]) if W is not free in E
- $\triangleright$ (λ W. F)[V/E]= λ W'. (F[W/W'][V/E]) if W' is free in F
- $\triangleright$ (F.F')[V/E]=(F[V/E])(F'[V/E])

β reduction rule

- The β conversion rule states that:
  ((λ V.E) E') == E[V/E']
  if all free occurences in E' remain free in E[V/E']

Example:
((λ n. n*4+5) 3)
$\Rightarrow$ (n*4+5)[3/n]
$\Rightarrow$ 3*4+5
$\Rightarrow$ 17

α conversion rule

- The α conversion rule states that:
  If V and W are varaibles
  E is a λ expression
  E[V/W] means the expression E with every free occurence of V in E replaced with W then:
  λ V.E == λ W.E[V/E]

Example:
λ V.(λ V. V)
=λ V.(λ W. W)

η conversion rule

It is the least frequently used rule

It is used for simplification

- ▷ The η conversion rule states that:
  ((λ V.E) V) == E
  if V is not free in E

Conclusion

- λ calclulus is the core of functional programming
- λ calclulus is equivalent to Turing machines
- For λ calclulus, if different evaluations orders do terminate, then the result will be the same

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