Computing and Software 701

Logic and Discrete Mathematics In Software Engineering

Fall 2004

Presentation Topics

Revised: 24 November 2004

1. Present a sound and complete natural deduction system for propositional logic. [TAKEN]
2. Present a sound and complete semantic tableau system for propositional logic. [TAKEN]
3. Present a sound and complete Gentzen system for propositional logic. [TAKEN]
4. Explain what the hyperreal numbers are. [TAKEN]
5. Explain what the surreal numbers are. [TAKEN]
6. Explain what the ordinals are. [TAKEN]
7. Present one of the set-theoretic paradoxes other than Russell's Paradox. [TAKEN]
8. Present a proof of the Schröder-Bernstein Theorem. [TAKEN]
9. Explain what a Boolean algebra is and give two examples of Boolean algebras. [TAKEN]
10. Present your solution of Exercise 9 of Exercise Group 1. [TAKEN]
11. Describe what "many-sorted" first-order logic is. [TAKEN]
12. Present a sound and complete natural deduction system for first-order logic. [TAKEN]
13. Present a sound and complete semantic tableau system for first-order logic. [TAKEN]

14. Present a sound and complete Gentzen system for first-order logic. [TAKEN]

15. Show that monadic first-order logic is decidable. That is, let $L = (\emptyset, \emptyset, \mathcal{P})$ where $\mathcal{P}$ is a finite set of unary predicate symbols, and then show that the problem of whether or not a formula of L is valid is decidable. [TAKEN]

16. Define the notion of an isomorphism between two models of a language of FOL. Explain what it means for two models to be "equal up to isomorphism". [TAKEN]

17. Present a sound and complete resolution proof system for first-order logic. Explain why proof systems of this kind are employed in automated theorem provers like Otter. [TAKEN]

18. Explain what logic programming is and briefly discuss how it is implemented in Prolog. [TAKEN]

19. Present the Knuth-Bendix completion procedure for transforming a finite set of equations into a "well-behaved" rewriting system. [TAKEN]

20. Present Church's lambda calculus. [TAKEN]

21. Present Curry's combinatory logic. [TAKEN]

22. Present Ackermann's function. [TAKEN]

23. Explain what Hilbert's ϵ operator is and show how it can be used to define the quantifiers $\forall$ and $\exists$.

24. Explain what group and ring homomorphisms are. [TAKEN]

25. Explain what a factor group is in group theory. [TAKEN]

26. Define what a module is in mathematics and give several examples of modules. [TAKEN]

27. Present the Knaster-Tarski fixed point theorem for complete partial orders. [TAKEN]

28. Present the fixed point theorem for continuous functionals.

29. Give a quick introduction to category theory. [TAKEN]

30. Present Dijkstra's shortest-path algorithm. [TAKEN]

31. Define what a *Goodstein sequence* is. Using ordinals show that every Goodstein sequences converges to 0! (See Reuben L. Goodstein, "On the restricted ordinal theorem", J. Symbolic Logic 9:33-41, 1944. L. Kirby and J. Paris showed in 1982 the remarkable result that the Goodstein theorem cannot be proven in first-order Peano arithmetic.)

32. Present Lakatos' method of proofs and refutations.