CAS 701 Fall 2004

01 What is Mathematics?

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Revised: 6 September 2004

Popular View

- The essence of mathematics is a huge body of concepts and facts about such things as time, measure, pattern, space, and logical consequence
- New concepts and facts are discovered by the definition-theorem-proof process
- Mathematics is infallible
  - Old concepts and facts are immutable
- Conjectures are either proved with a proof or refuted with a counterexample

Mathematics-as-Process View

The essence of mathematics is a process consisting of three intertwined activities:

1. **Model creation**: Mathematical models representing mathematical aspects of the world are created

2. **Model exploration**: The models are explored by:
   a. Stating and proving conjectures
   b. Performing calculations
   c. Creating and studying visual representations

3. **Model connection**: Models with similar structure are connected to each other to facilitate the creation and exploration of new models

Proof

- Mathematical proof is an essential component of the mathematics process which is unique to mathematics
- It is a method of **justification**, **communication**, and **discovery**
- An **informal proof** is a convincing argument that a statement about a model is true
- A **formal proof** is a logical deduction from a set of premises to a conclusion
  - Can be mechanically checked
- A formal proof can be presented in two ways:
  - As a **description** of the actual deduction
  - As a **prescription** for creating the deduction

Lakatosian View

- Presented by **Imre Lakatos** in *Proofs and Refutations*, Cambridge University Press, 1976

- Mathematical reasoning is dialectical
  - Dialectic between a theory and it theorems
  - Dialectic between a conjecture and its proof

- New mathematics is discovered by analyzing the proofs of conjectures according to the **method of proof and refutations**

- The definition-theorem-proof style of presentation hides the true nature of mathematics

Mathematics for Software Engineering

Mathematics enters software engineering from three sources:

- Mathematics underlying the concepts and techniques of **computer science**
  - This is largely **discrete mathematics**
- Mathematics used to manage the **software development process**
  - Many of the principal products of software engineering are essentially mathematical models (e.g., requirement specifications, design documents, software descriptions, and software code)
  - This is largely **logic**
- Mathematics needed to understand **physical devices**
  - This is largely **continuous mathematics**

Software Development as Mathematics

- A rigorous software development process is a special case of the mathematics process
  - Formulation of software requirements and design and implementation of software involves the **creation of mathematical models**
  - Verification and analysis of software involves the **exploration of mathematical models**
  - Reuse of software involves the **connection of mathematical models**

- Managing software documentation is a special case of managing mathematical knowledge

The Need for Mechanization

- The mathematical models produced by software engineering are often different than traditional mathematical models:
  - They tend to be very large with many details
  - They are usually produced by teams of developers
  - Most of the content is mathematically uninteresting
  - Small mistakes can have a dire effect on such things as safety, cost, and security

- The traditional paper-based mathematics practice is becoming increasingly untenable for software development

- The mathematics part of the software development process must be mechanized