Computing and Software 701

Logic and Discrete Mathematics In Software Engineering

Fall 2005

Exercise Group 1

100 pts.

Due October 12, 2005

Revised: 26 September 2005

In the following exercises, let **H** be the Hilbert-style proof system for the propositional language L_0 presented in class.

1. [12 pts.] Exercise 24 on p. 18 of Rosen.
2. [4 pts.] Exercise 52 on p. 20 of Rosen.
3. [4 pts.] Exercise 54 on p. 20 of Rosen.
4. [8 pts.] Exercise 56 on p. 20 of Rosen.
5. [8 pts.] Exercise 8 on p. 26 of Rosen.
6. [4 pts.] Exercise 48 on p. 28 of Rosen.
7. [6 pts.] Exercise 54 on p. 28 of Rosen.
8. [6 pts.] Assuming that $\{\neg, \Rightarrow\}$ is a complete set of propositional connectives, show that the set $\{\,|\,\}$ of just the Sheffer stroke is also complete.
9. [12 pts.] Define what it means for a formula of propositional logic to be in *conjunctive normal form* and in *disjunctive normal form*. Let L be a language of propositional logic with the connectives $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$. Write an algorithm that, given a formula A of L as input, returns a formula A' as output such that A' is in conjunctive normal form and $A \Leftrightarrow A'$ is a tautology.
10. [12 pts.] Construct a proof of $p_0 \Rightarrow \neg\neg p_0$ in **H**.

11. [12 pts.] Prove that **H** is sound.

12. [12 pts.] Let Σ be a set of formulas of L_0. Assuming that **H** is complete, prove that Σ is consistent in **H** iff Σ is satisfiable.