

**Computing and Software 701**  
**Logic and Discrete Mathematics**  
**In Software Engineering**  
**Fall 2005**

**Exercise Group 2**

**100 pts.**

**Due 26 October 2005**

Revised: 12 October 2005

1. [4 pts.] Exercise 18 on p. 109 of Rosen.
2. [4 pts.] Exercise 36 on p. 110 of Rosen.
3. [4 pts.] Exercise 14 on p. 495 of Rosen.
4. [4 pts.] Exercise 10 on p. 513 of Rosen.
5. [4 pts.] Exercise 28 on p. 42 of Rosen.
6. [4 pts.] Exercise 46 on p. 43 of Rosen.
7. [16 pts.] Exercise 12 on p. 52 of Rosen.
8. [8 pts.] Exercise 50 on p. 56 of Rosen.
9. [8 pts.] Exercise 32 on p. 237 of Rosen.
10. [4 pts.] Exercise 40 on p. 238 of Rosen.
11. [8 pts.] Let  $f : A \rightarrow B$  and  $g : B \rightarrow C$  be total, and let  $h = g \circ f : A \rightarrow C$  be the composition of  $g$  and  $f$ .
  - (a) Prove that, if  $f$  and  $g$  are injective, then  $h$  is injective, but the converse is false.
  - (b) Prove that, if  $f$  and  $g$  are surjective, then  $h$  is surjective, but the converse is false.

12. [8 pts.] What is the cardinality of the function space  $\mathbf{N} \rightarrow \mathbf{N}$ , where  $\mathbf{N}$  denotes the set of natural numbers?
13. [8 pts.] Define what it means for a formula of FOL to be in *prenex normal form*. Let  $L$  be a language of FOL. Write an algorithm that, given a formula  $A$  of  $L$  as input, returns a formula  $A'$  as output such that  $A'$  is in prenex normal form and  $A \Leftrightarrow A'$  is valid.
14. [8 pts.] Use the compactness theorem for FOL to show that every FOL theory that has arbitrarily large finite models has an infinite model. Is there an FOL theory of all the finite models for a language  $L$ ?
15. [8 pts.] Express the theory of vector spaces as a theory of FOL.