Computing and Software 701

Logic and Discrete Mathematics In Software Engineering

Fall 2004

Midterm Test

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You have 90 minutes to complete this test consisting of 1 page and 6 questions. Write your answers in the examination booklet provided to you. Give reasons for your answers. Good luck!

(1) Consider the formula

$$((A \vee B) \Rightarrow C) \Rightarrow ((\neg B \wedge A) \Rightarrow \neg C)$$

of propositional logic. Is this formula valid, satisfiable but not valid, or not satisfiable?

(2) Suppose $(A, \leq_1)$ and $(B, \leq_2)$ are partial orders. Consider $(A \times B, \leq_3)$ where, for all $x_1, x_2 \in A$ and $y_1, y_2 \in B$, $(x_1, y_1) \leq_3 (x_2, y_2)$ iff $x_1 \leq_1 x_2$ or $y_1 \leq_2 y_2$. Can we infer that $(A \times B, \leq_3)$ is a partial order? If so, prove that it is, and if not, give a counterexample.

(3) Let $\mathbf{H}$ be the Hilbert-style proof system for the language L_0 of propositional logic given in class. Prove that the converse of the deduction theorem holds for L_0, i.e., that $\Sigma \vdash_{\mathbf{H}} A \Rightarrow B$ implies $\Sigma \cup \{A\} \vdash_{\mathbf{H}} B$.

(4) Let $f : A \to B$ and $g : B \to C$ be total, and let $h = g \circ f : A \to C$ be the composition of g and f. Prove that, if f and g are injective, then h is injective.

(5) Let S be the set of all total functions $f : \{\text{T}, \text{F}\} \to \mathbf{N}$. Prove that S is equipollent to $\mathbf{N}$, or prove that S is equipollent to $\mathbf{R}$.

(6) Let $L = (\emptyset, \emptyset, \{=, p\})$ be a language of FOL where p is binary. Find a set Γ of formulas of FOL such that, for each model $M = (D, I)$ of $T = (L, \Gamma)$, $I(p)$ partitions D and each member of the partition contains at least two elements.

Test ends here.