Computing and Software 701

Logic and Discrete Mathematics
In Software Engineering

Fall 2008

Exercise 2

100 pts.

Due 16 October 2008

Revised: 8 October 2008

In the following exercises, Rosen means the textbook K. H. Rosen, *Discrete mathematics and its Applications, Fifth Edition*, 2003.

1. [4 pts.] State and prove the de Morgan Laws for sets.

2. [4 pts.] Exercise 16 on p. 95 of Rosen.

3. [4 pts.] Exercise 28 on p. 95 of Rosen.

4. [4 pts.] Exercise 18 on p. 109 of Rosen.

5. [4 pts.] Exercise 36 on p. 110 of Rosen.

6. [10 pts.] Exercise 14 on p. 495 of Rosen.

7. [4 pts.] Exercise 10 on p. 513 of Rosen.

8. [8 pts.] Exercise 28 on p. 514 of Rosen.

9. [8 pts.] Exercise 32 on p. 237 of Rosen.

10. [4 pts.] Exercise 40 on p. 238 of Rosen.

11. [4 pts.] Show how a relation $R \subseteq A \times B$ can be transformed into an "equivalent" total function $f_R : A \to \mathcal{P}(B)$, where $\mathcal{P}(B)$ is the power set of B. (A function like f_R is sometimes called a *many-valued function*.)

12. [10 pts.] What is the cardinality of the function space $\mathbf{N} \to \mathbf{N}$, where $\mathbf{N}$ denotes the set of natural numbers?

13. [16 pts.] Let $f : A \to B$ and $g : B \to C$ be total, and let $h = g \circ f :$ $A \to C$ be the composition of g and f.

    (a) Prove that, if f and g are injective, then h is injective, but the converse is false.

    (b) Prove that, if f and g are surjective, then h is surjective, but the converse is false.

14. [16 pts.]

    (a) Let T_n be a full binary tree of height $n \geq 1$. What is the cardinality of the set of nodes in T_n? What is the cardinality of the set of paths in T_n?

    (b) Let T_∞ be a full binary tree of infinite height. What is the cardinality of the set of nodes in T_∞? What is the cardinality of the set of paths in T_∞?