Computing and Software 701

Logic and Discrete Mathematics In Software Engineering

Fall 2008

Exercise 3

100 pts.

Due 6 November 2008

Revised: 17 October 2008

In the following exercises, Rosen means the textbook K. H. Rosen, *Discrete mathematics and its Applications, Fifth Edition*, 2003.

1. [8 pts.] Exercise 26 on pp. 528–9 of Rosen.
2. [8 pts.] Exercise 28 on p. 529 of Rosen.
3. [4 pts.] Exercise 28 on p. 42 of Rosen.
4. [4 pts.] Exercise 46 on p. 43 of Rosen.
5. [16 pts.] Exercise 12 on p. 52 of Rosen.
6. [4 pts.] Exercise 50 on p. 56 of Rosen.
7. [8 pts.] Suppose $(S_1, \leq_1)$ and $(S_2, \leq_2)$ are partial orders. Prove that $(S_1 \times S_2, \leq)$ is a partial order where $(s_1, s_2) \leq (s_1', s_2')$ iff $s_1 \leq_1 s_1'$ and $s_2 \leq_2 s_2'$.
8. [8 pts.] Prove that a total order is a lattice.
9. [10 pts.] Define what it means for a formula of FOL to be in *prenex normal form*. Let L be a language of FOL. Write an algorithm that, given a formula A of L as input, returns a formula A' as output such that A' is in prenex normal form and $A \Leftrightarrow A'$ is valid.
10. [10 pts.] Express the theory of lattices as a theory of FOL. This can be done in two ways: (1) where partial order ($\leq$) is primitive and (2) where meet ($\vee$) and join ($\wedge$) are primitive.

11. [10 pts.] Express the theory of vector spaces as a theory of FOL.

12. [10 pts.] Use the compactness theorem for FOL to show that every FOL theory that has arbitrarily large finite models has an infinite model. Is there an FOL theory of all the finite models for a language L?