Computing and Software 701

Logic and Discrete Mathematics In Software Engineering

Fall 2008

Exercise 4

100 pts.

Due 4 December 2008

Revised: 28 November 2008

In the following exercises, Rosen means the textbook K. H. Rosen, *Discrete mathematics and its Applications, Fifth Edition*, 2003.

1. [8 pts.] Let $T = (L, \Gamma)$ be a theory of groups in FOL where

$$L = (\{e\}, \{\mathsf{mul}, \mathsf{inv}\}, \{=\})$$

with mul binary and inv unary and Γ is the set of the following formulas of L:

   (a) $\forall x, y, z \,.\, x \;\mathsf{mul}\; (y \;\mathsf{mul}\; z) = (x \;\mathsf{mul}\; y) \;\mathsf{mul}\; z$.

   (b) $\forall x \,.\, x \;\mathsf{mul}\; e = x$.

   (c) $\forall x \,.\, e \;\mathsf{mul}\; x = x$.

   (d) $\forall x \,.\, x \;\mathsf{mul}\; \mathsf{inv}(x) = e$.

   (e) $\forall x \,.\, \mathsf{inv}(x) \;\mathsf{mul}\; x = e$.

   Construct a term rewriting system that is sound and complete with respective to T, finite, confluent, and finitely terminating.

2. [8 pts.] Let L be a language of propositional logic with the connectives $\neg, \wedge, \vee$. Construct a rewriting system that is finite and finitely terminating and that reduces any formula A of L to an equivalent formula in disjunctive normal form.

3. [8 pts.] Prove that a term rewriting system is Church-Rosser iff it is confluent.

4. [4 pts.] Exercise 14 on p. 253 of Rosen.
5. [4 pts.] Exercise 16 on p. 253 of Rosen.
6. [4 pts.] Exercise 36 on p. 272 of Rosen.
7. [4 pts.] Exercise 44 on p. 272 of Rosen.
8. [4 pts.] Exercise 48 on p. 273 of Rosen.
9. [4 pts.] Exercise 52 on p. 273 of Rosen.
10. [18 pts.] Show that the following functions are primitive recursive:
    (a) Addition.
    (b) Multiplication.
    (c) Exponentiation.
11. [8 pts.] Let L_0 be the propositional language defined on slide 4 of the 02 Propositional Logic slides. For a formula φ of L_0, let $p(\varphi)$ be the number of distinct propositional symbols occurring in φ and $i(\varphi)$ be the number of implication symbols occurring in φ. Prove by structural induction that, for all formulas φ of L_0, $p(\varphi) \leq i(\varphi) + 1$.
12. [4 pts.] Give a natural example of a well-founded relation that is not a partial order.
13. [4 pts.] Show that Ackermann's function is an instance of well-founded recursion.
14. [18 pts.] Let $f : \mathbf{N} \to \mathbf{N}$ generate the Fibonacci sequence.
    (a) Show that f is a primitive recursive function.
    (b) Define f by well-founded recursion.
    (c) Define f by recursion via a monotone functional.