Resolution Proof Systems

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Conjunctive Normal Form

CNF = Conjunction of disjunctions

$$\underbrace{(a1 \vee a2 \vee \ldots)}_{\text{Clause}} \wedge (b1 \vee b2 \vee \ldots) \wedge \ldots$$

Resolution Proving: Introduction 1

Given : $x, x \Rightarrow y$
Conclude: y

1) Modus Ponens: $$\frac{(x, x \Rightarrow y)}{y}$$

2) Without MP:

$$x \wedge x \Rightarrow y$$
$$x \wedge (\neg x \vee y)$$
$$y$$

x is true, so ~x is false. For (~x v y) to be true, y must be true. Therefore, y.

Resolution Proving: Introduction 2

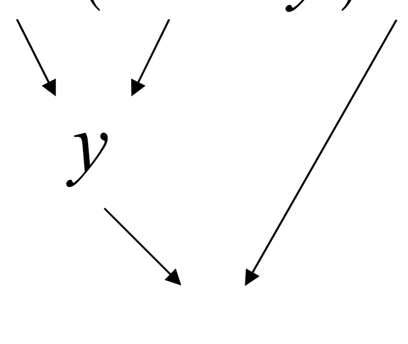
Given : $x \, , x \Rightarrow y$

Conclude: y

3) Resolution:

Negate the conclusion and add it as an argument:

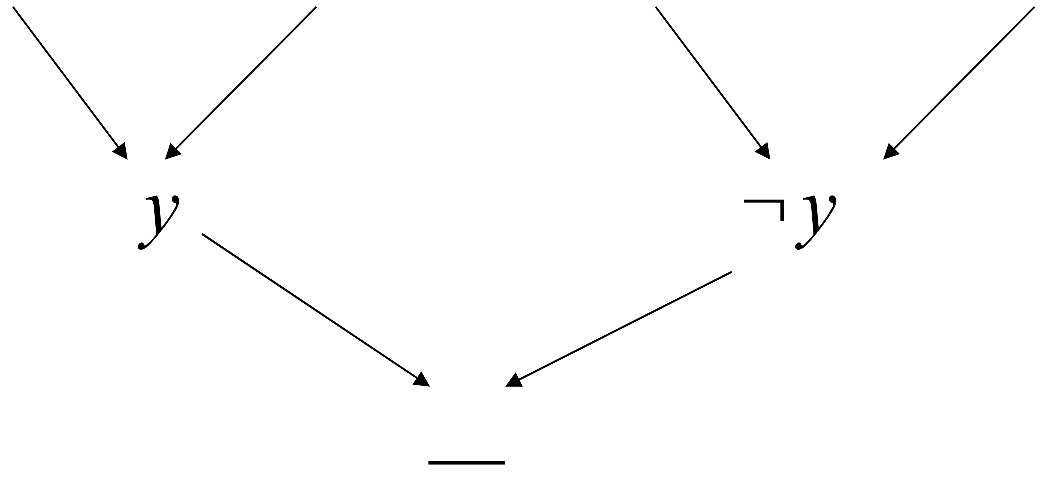
$$x \wedge (\neg x \vee y) \wedge \neg y$$



Resolution Proving: Introduction 3

Resolve:

$$(x \vee y) \wedge (\neg x \vee y) \wedge (x \vee \neg y) \wedge (\neg x \vee \neg y)$$



Resolution Proof Rule 1

C1, C2 = Clause $(a1 \vee a2 \vee \ldots)$
Z = Term

$$\frac{((C1 \vee Z) \wedge (C2 \vee \neg Z))}{(C1 \vee C2)}$$

Resolution Proof Rule 2

Res. $(\neg x, x) = F$

Res. $(x \vee y, \neg x) = y$

Res. $(x \vee \neg y, \neg x \vee y) = (x \vee \neg x)$

Only one term removed per application.

Res. $(x \vee y, y \vee \neg z) = \emptyset$

There is nothing to resolve.

Clausal Form

FOL formulae must be converted to C.F:

1. Eliminate => and <=>.
2. Move ~ inward. (DeMorgan's, etc.)
3. Rename variables, if two quantifiers have the same bound variable name.
4. Eliminate existential quantifiers using Skolem functions. (Next slide.)
5. Move all universal quantifiers to the left.
6. Transform what remains into CNF.

Skolemization

Skolemization removes an existential quantifier from a term by replacing it with a Skolem fn:

$$\exists_x \forall_y P(x, y) \Rightarrow \forall_y P(\underline{c}, y)$$

$$\forall_x \exists_y P(x, y) \Rightarrow \forall_x P(x, \underline{f(x)})$$

$$\forall_x \forall_y \exists_z P(x, y, z) \Rightarrow \forall_x \forall_y P(x, y, \underline{f(x, y)})$$

$$\forall_x \exists_y \forall_z P(x, y, z) \Rightarrow \forall_x \forall_z P(x, \underline{f(x)}, z)$$

Soundness & Completeness 1

Resolution is sound:

$$\vdash Res(C1, C2) = Q \Rightarrow \{C1, C2\} \models Q$$

But, it isn't complete for direct inference:

$$A \models A \vee B \Rightarrow \vdash Res(A, ?) \neq A \vee B$$

$$Res(A, ?) = \emptyset$$

Soundness & Completeness 2

With a complete search algorithm, it's complete for proofs by counter-example:

$$\hbar \models \bot \Rightarrow \vdash Res \; \bot$$

(Proofs are beyond the scope of the presentation.)

Resolution & Automated Proving

Is popular for automated proofs because of it's clarity and simplicity.

Hilbert-style proof systems aren't good for AP because an infinite number of axioms may be generated.

Resolution has only one rule: Resolution.

Thank You

Thank you for listening.

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