CAS 734 Winter 2005

01 What is Formalized Mathematics?

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Revised: 3 January 2005

Basic Questions

1. What is formalized mathematics?

2. Why is formalized mathematics useful?

3. How is formalized mathematics done?

What is Formalized Mathematics?

- **Formalized mathematics** is conventional mathematics that is expressed and developed within a formal logic

  - What is mathematics?
  - What is a formal logic?

- Formalized mathematics emphasizes the "mathematics", while **formal mathematics** emphasizes the "formality"

- Before the invention of the modern computer, formalized mathematics was of only theoretical interest

  - Biggest precomputer development was Whitehead and Russell's **Principia Mathematica** (1910–1913)

Why is Formalized Mathematics Useful?

- Formalized mathematics can be mechanized with the help of logic and the computer

  - Complexity can be better managed
  - Deduction can be machine checked
  - Thus mathematics can be made easier to do
  - Thus results can be more reliable

- Formalized mathematics can be disseminated and accessed on the Web

  - Mathematics can be stored as dynamic information
  - Mathematical entities can be reused in multiple contexts

- Complex systems, such as software systems, can be represented and analyzed as mathematical models

How is Formalized Mathematics Done?

- Mathematics is performed with the assistance of a mechanized mathematics system
  - What is a mechanized mathematics system?

- Mathematics is organized using the axiomatic method
  - What is the axiomatic method?

- Axiomatic theories are developed
  - What are the methods for creating, exploring, connecting axiomatic theories?

- Conjectures are formally proved
  - What are the techniques of formal proof?