Computing and Software 734
Formalized Mathematics
Fall 2006

Exercise 7
50 pts.
Due 11 December 2006

Assigned: 13 November 2006
Revised: 13 November 2006

Part 1

Starting from scratch, create an IMPS theory of Peano arithmetic called `peano1` with the constants 0, S, $+$, and $*$ and the following axioms:

1. 0 is not a successor.
2. S is injective.
3. Induction.
4. The two axioms that specify $+$ recursively.
5. The two axioms that specify $*$ recursively.

Part 2

Prove the following theorems in `peano1`:

1. 0 is the additive identity.
2. $S(0)$ is the multiplicative identity.
3. $+$ is associative.
4. $*$ is associative.
5. $+$ is commutative.
6. $*$ is commutative.

Part 3

Create a compound macete that reduces any ground expression of `peano1` (i.e., any expression containing no variables and no constants other than 0, S, $+$, and $*$) to an expression of the form $S^m(0)$ (i.e., S applied to 0 m times). Give several examples expressed as theorems to show that it works.

Part 4

Create the following three interpretations:

1. An interpretation of `peano1` in `h-o-real-arithmetic`.
2. An interpretation of `commutative-monoid-theory` in the additive part of `peano1`.
3. An interpretation of `commutative-monoid-theory` in the multiplicative part of `peano1`.