

Computing and Software 734
Formalized Mathematics

William M. Farmer
McMaster University, Winter 2014

Exercise 6

20 pts.

Due 17 March 2014

Assigned: 7 February 2014

Revised: 17 March 2014

Part 1. Starting from scratch, create a theory of (second-order Peano arithmetic) named something like `PeanoArithmetic`. It should include constants 0 , S , $+$, and $*$ and the following axioms:

1. 0 is not a successor.
2. S is injective.
3. Induction.
4. The two axioms that specify $+$ recursively.
5. The two axioms that specify $*$ recursively.

Prove the axioms of a commutative semiring in `PeanoArithmetic`:

1. $+$ is associative.
2. $*$ is associative.
3. $+$ is commutative.
4. $*$ is commutative.
5. 0 is the additive identity.
6. $S(0)$ is the multiplicative identity.
7. $*$ distributes over $+$.

8. $0 * x = 0$ for all x .

You will probably have to prove a few additional intermediate lemmas.

Part 2. Create an alternate new version of `PeanoArithmetic` named something like `PeanoArithmeticAlt` in which `+` and `*` are defined constants rather than primitive constants specified by axioms. Prove the axioms of a commutative semiring in `PeanoArithmeticAlt`.

Send the instructor the files you produced with comments and instructions on how they can be loaded and checked.