System Modelling using Event-B

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Outline

1. Model Development
2. Machines and Contexts
3. Event
4. Action
5. Proof Obligation (POs)
6. Refinement
7. Tools and Books
8. Case Studies

Model Development with Event-B

Event-B

- is designed by J.-R. Abrial in 1988.
- is not a programming language.
- notations are based-on set-theory and first-order logic.
- notations are used to develop the mathematical models of discrete transition systems (system behaviour).
- supports refinement based development.
- models can be developed using the Rodin Platform.

Modelling Systems

1. Problem Understanding
2. Identifying and organizing the requirements and properties
3. To specify a very abstract model
4. Adding new requirements using small refinement steps
5. Stop the refinement if model is enough concrete

Set-theoretical Notations

Name	Syntax	Definition
Binary relation	$s \leftrightarrow t$	$(\mathcal{P})(s \times t)$
Composition of relations	$r_1 \ ; \ r_2$	$\{x, y \mid x \in a \ \wedge \ y \in b \wedge$ $\exists z.(z \in c \ \wedge \ x, z \in r_1 \ \wedge \ z, y \in r_2\}$
Inverse relation	r^{-1}	$\{x, y \mid x \in \mathcal{P}(a) \ \wedge \ y \in \mathcal{D}(b) \ \wedge \ a \mapsto b \in r)\}$
Domain	$dom(r)$	$\{a \mid a \in s \ \wedge \ \exists b.(b \in t \ \wedge \ a \mapsto b \in r)\}$
Range	$ran(r)$	$dom(r^{-1})$
Identity	$id(s)$	$\{x, y \mid x \in s \ \wedge \ y \in s \ \wedge \ x = y\}$
Restriction	$s \lhd r$	$id(s); r$
Co-restriction	$r \rhd s$	$r; id(s)$
Anti-restriction	$s \lhd\!\!\!- r$	$(dom(r) - s) \lhd r$
Anti-co-restriction	$r \rhd\!\!\!- s$	$r \rhd (ran(r) - s)$
Image	$r[w]$	$ran(w \lhd r)$
Overriding	$q \lhd\!\!\!+ r$	$(\mathrm{dom}(r) \lhd q) \cup r$
Partial function	$s \nrightarrow t$	$\{r \mid r \in s \leftrightarrow t \ \wedge \ (r^{-1}; r) \subseteq id(t)\}$

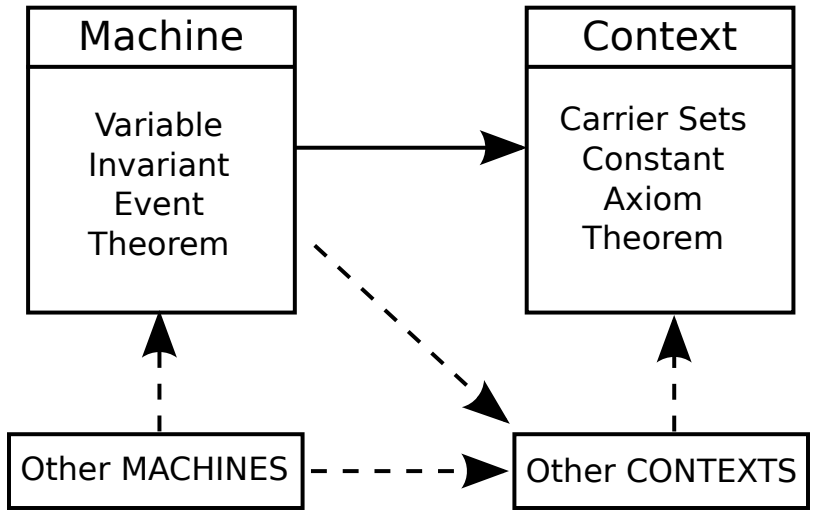
Machines and Contexts

Contexts

Contexts are used to formalize the static structure of a discrete system (constants and axioms)

Machines

Machines are used to formalize the dynamic structure of a discrete system (variables, invariants, and events).



Context Structure

context
< *context identifier* >
extendes
< *context identifier* >
sets
< *set identifier* >
constants
< *constant identifier* >
axioms
< *label* >:< *predicate* >
theorems
< *label* >:< *predicate* >
end

context
c1
sets
S
constants
x
y
axioms
axm1: $x \in \mathbb{N}$
axm2: $y \in 1..x \rightarrow S$
theorems
thm1: $x \in \mathbb{N}1$
end

Machine Structure

machine
< *machine identifier* >
refines
< *machine identifier* >
sees
< *context identifier* >
variables
< *variable identifier* >
invariants
< *label* >:< *predicate* >
theorems
< *label* >:< *predicate* >
events
< *initialisation* >< *evn* 1...*evn n* >
variant
< *variant* >
end

machine
m1
sees
c1
variables
i
invariants
axm1: $i \in 1..x$
events
...
end

Event Structure

$< \textit{event identifier} > \triangleq$
status
$\{\textit{ordinary}, \textit{convergent}, \textit{anticipated}\}$
refines
$< \textit{event identifier} >$
any
$< \textit{parameter identifier} >$
where *or* **when**
$< \textit{label} >: < \textit{predicate} >$
with
$< \textit{label} >: < \textit{witness} >$
then
$< \textit{label} >: < \textit{action} >$
end

Event *Update*
refines
U*pdate*
any
a
where
grd1: $a \in 1..x$
then
act1: $i := a$
end

Events

Event: E	**Before-After Predicate**
BEGIN $x : \mid P(x, x')$ END	$P(x, x')$
WHEN $G(x)$ THEN $x : \mid P(x, x')$ END	$G(x) \wedge P(x, x')$
ANY t WHERE $G(t, x)$ THEN $x : \mid P(t, x, x')$ END	$\exists t.(G(t, x) \wedge P(t, x, x'))$

Event: E	**Guard:** $grd(E)$
BEGIN $x : \mid P(x, x')$ END	$TRUE$
WHEN $G(x)$ THEN $x : \mid P(x, x')$ END	$G(x)$
ANY t WHERE $G(t, x)$ THEN $x : \mid P(t, x, x')$ END	$\exists t.G(t, x)$

Actions

Actions

An action modifies one or more state variables. There are two types of actions: deterministic and non-deterministic

Deterministic Action

$x := p + q$

$y := max(p, q)$

Non-Deterministic Action

$x : |x' = p + q$

$x, y : |x' > x \wedge y' < x'$

$x :\in \{p + 1, q + 3, r + 4\}$

$x : |x' \in \{p + 1, q + 3, r + 4\}$

Proof Obligation

The POs are automatically generated by a Rodin Platform tool called the Proof Obligation Generator. A list of POs is given as follows:

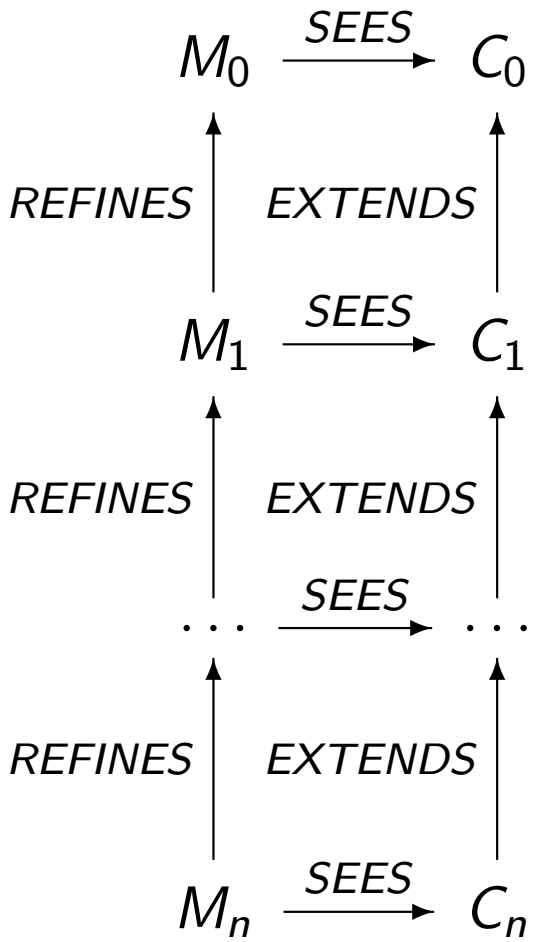
- Well-definedness (WD)
- Invariant preservation (INV)
- Non-deterministic action feasibility (FIS)
- Guard strengthening(GRD)
- Simulation (SIM)
- Numeric variant (NAT)
- Proving theorems (THM)

Proof Obligations

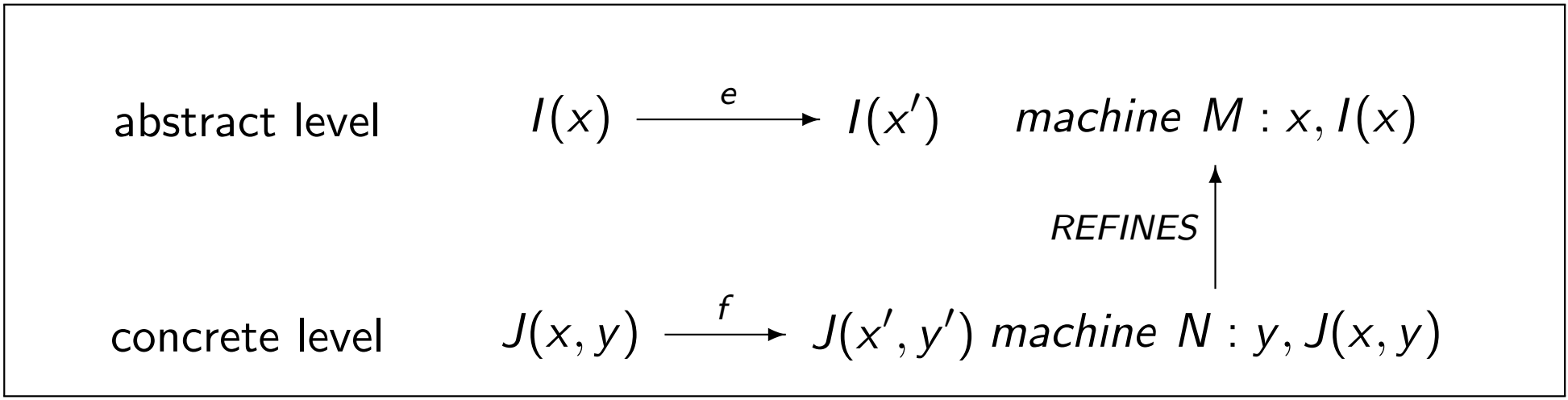
INV1	$A(s,c) \wedge G(x,s,c,v) \wedge BA(e)(x,s,c,v,v')$ $\vdash$ $I(s,c,v')$
INV2	$A(s,c) \wedge I(s,c,v) \wedge G(x,s,c,v) \wedge BA(e)(x,s,c,v,v')$ $\vdash$ $I(s,c,v')$
FIS	$A(s,c) \wedge I(s,c,v) \wedge G(x,s,c,v)$ $\vdash$ $\exists v'.BA(e)(x,s,c,v,v')$
GRD	$A(s,c) \wedge I(s,c,v) \wedge J(s,c,v,w) \wedge H(y,s,c,w) \wedge Wit(x,y,s,c,w)$ $\vdash$ $grd(x,s,c,v)$
DEAD	$A(s,c) \wedge I(s,c,v)$ $\vdash$ $(grd(g_1) \vee ... \vee grd(g_n))$

Refinement

- To add more details (like superposition).
- To introduce a set of new events, and safety properties.
- To prove that the concrete behaviors are abstract ones.
- Each new event refines SKIP.
- No deadlock



Key Steps of the Refinement



- An event f simulates an event e.
- f modifies y
- e modifies x
- If e preserves $I(x)$ and f simulates e, then f preserves $I(x)$

$$I(x) \wedge J(x, y) \wedge BA(f)(y, y') \Rightarrow \exists x'.(BA(e)(x, x') \wedge J(x', y'))$$

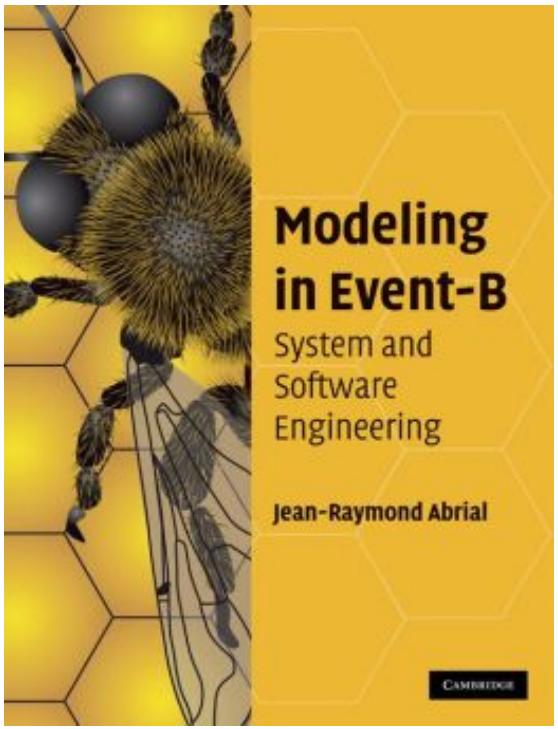
The invariant in the refinement model is preserved by the refined event and the activation of the refined event triggers the corresponding abstract event.

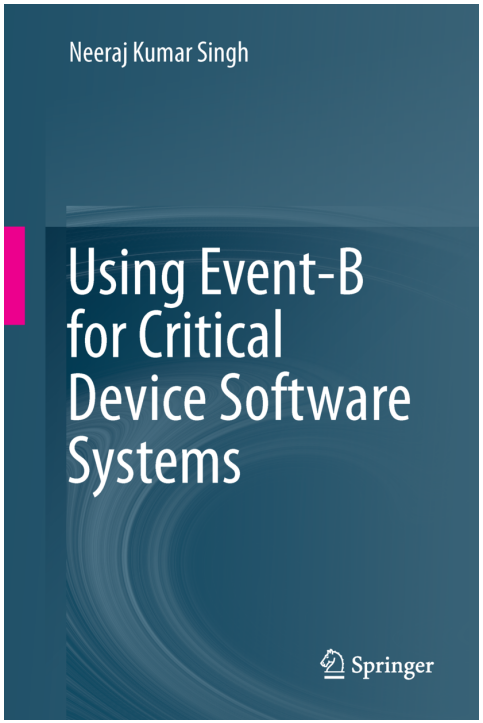
Proof Obligations for Refinement

REF1	$InitC(y)$ $\Rightarrow$ $\exists x.(InitA(x) \wedge J(x, y)$
REF2	$I(x) \wedge J(x, y) \wedge BAC(y, y')$ $\Rightarrow$ $\exists x.(BAA(x, x') \wedge J(x', y')$
REF3	$I(x) \wedge J(x, y) \wedge BAC(y, y')$ $\Rightarrow$ $J(x, y')$
REF3	$I(x) \wedge J(x, y) \wedge (G_1(x) \vee ... \vee G_n(x))$ $\Rightarrow$ $H_1(y) \vee ... \vee H_k(y)$

Tools and Books

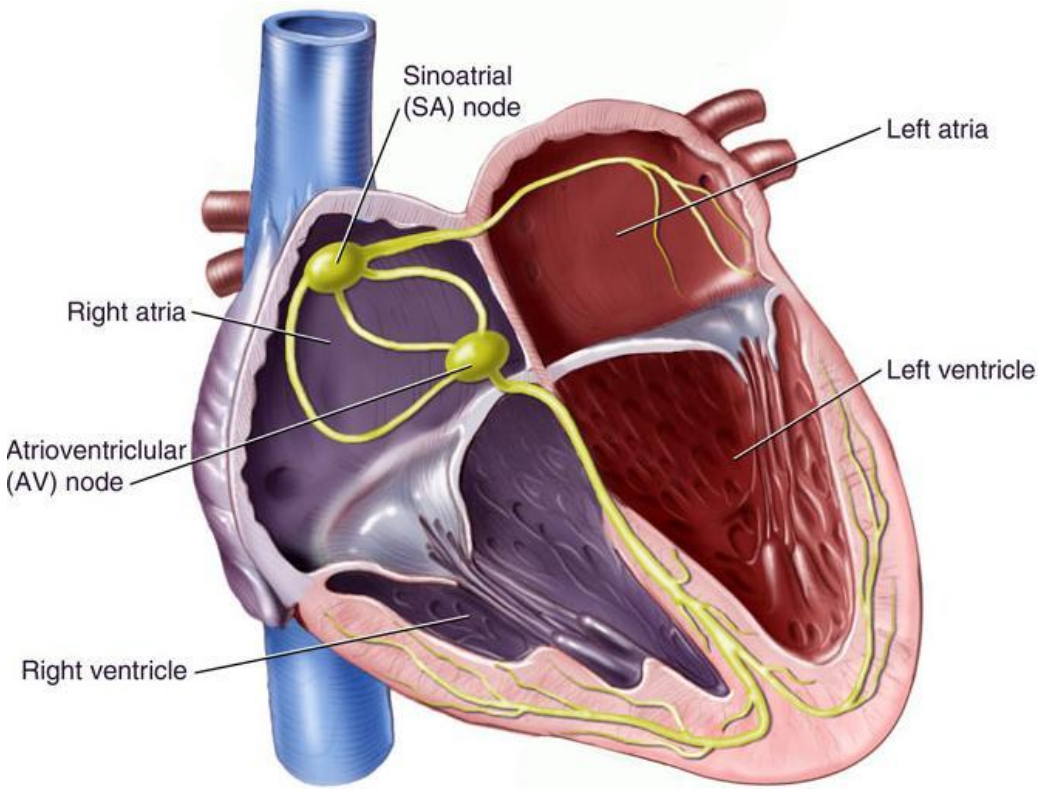
- RODIN Platform: http://www.event-b.org/platform.html
- Event B: http://www.event-b.org
- ProB :http://www.stups.uni-duesseldorf.de/ProB
- EB2ALL Toolset: http://eb2all.loria.fr



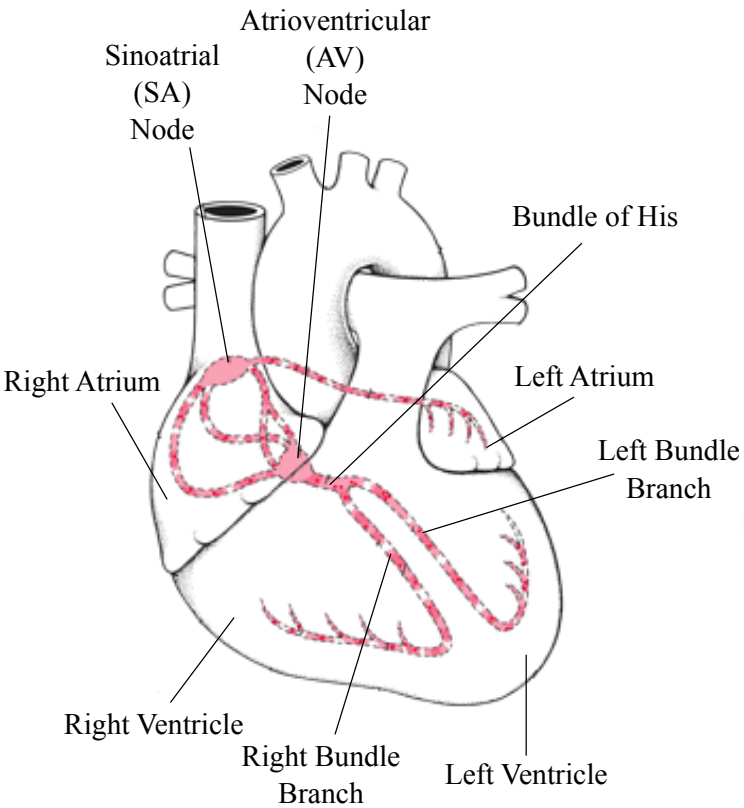


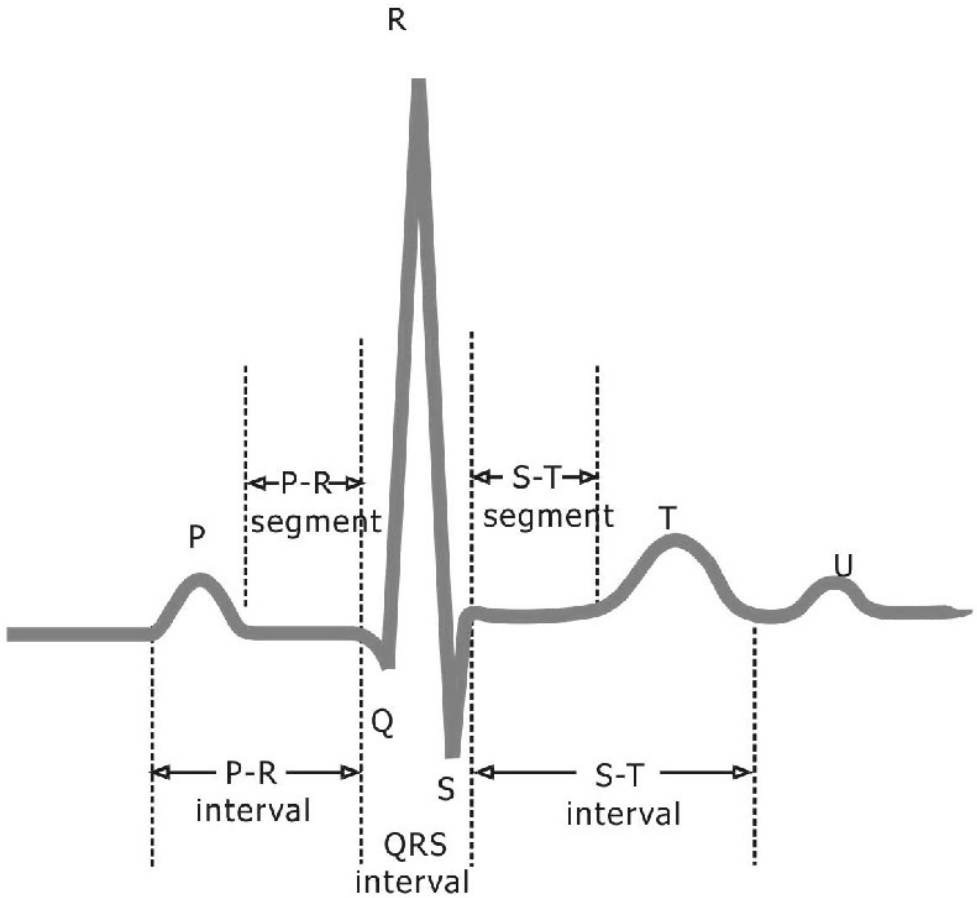
Cardiac Pacemaker

Heart System



Electrical Signal of the Heart





The Cardiac Pacemaker

Pacemaker

A pacemaker is an electronic device implanted in the body to regulate the abnormal heart rhythm (bradycardia).
Type of pacemakers:1,2 and 3-Electrodes.

The Cardiac Pacemaker

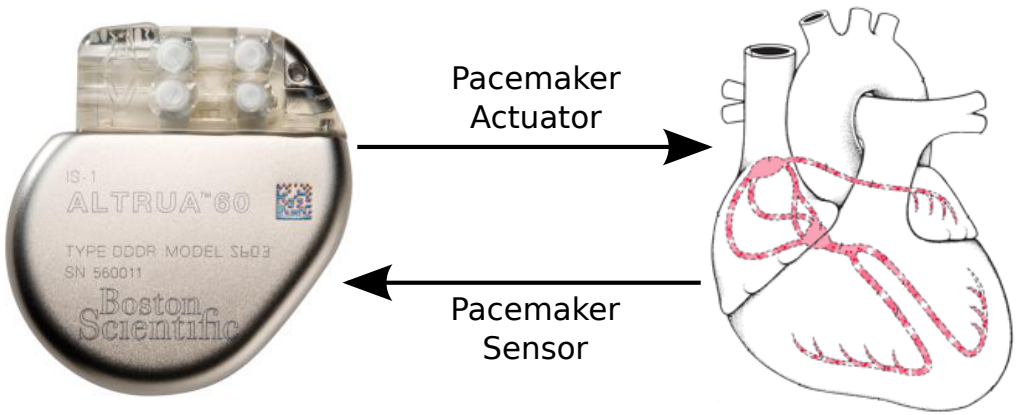
Operating Modes : NASPE/BPEG Generic Code

Category	**Chambers Paced**	**Chambers Sensed**	**Response to Sensing**	**Rate Modulation**
Letters	**O**-None **A**-Atrium **V**-Ventricle **D**-Dual(A+V)	**O**-None **A**-Atrium **V**-Ventricle **D**-Dual(A+V)	**O**-None **T**-Triggered **I**-Inhibited **D**-Dual(T+I)	**R**-Rate Modulation

i.e. AOO, VOO, AAI, AAT, VVI, VVT, AATR,VVTR, AOOR etc. . .
Periodic stimuli : (AOO, VOO and DOO)
Aperiodic stimuli : (AAI, VVI, DDD, DDI, etc.)

The Cardiac Pacemaker

- Simple Pacemaker Model
- Closed-loop Model



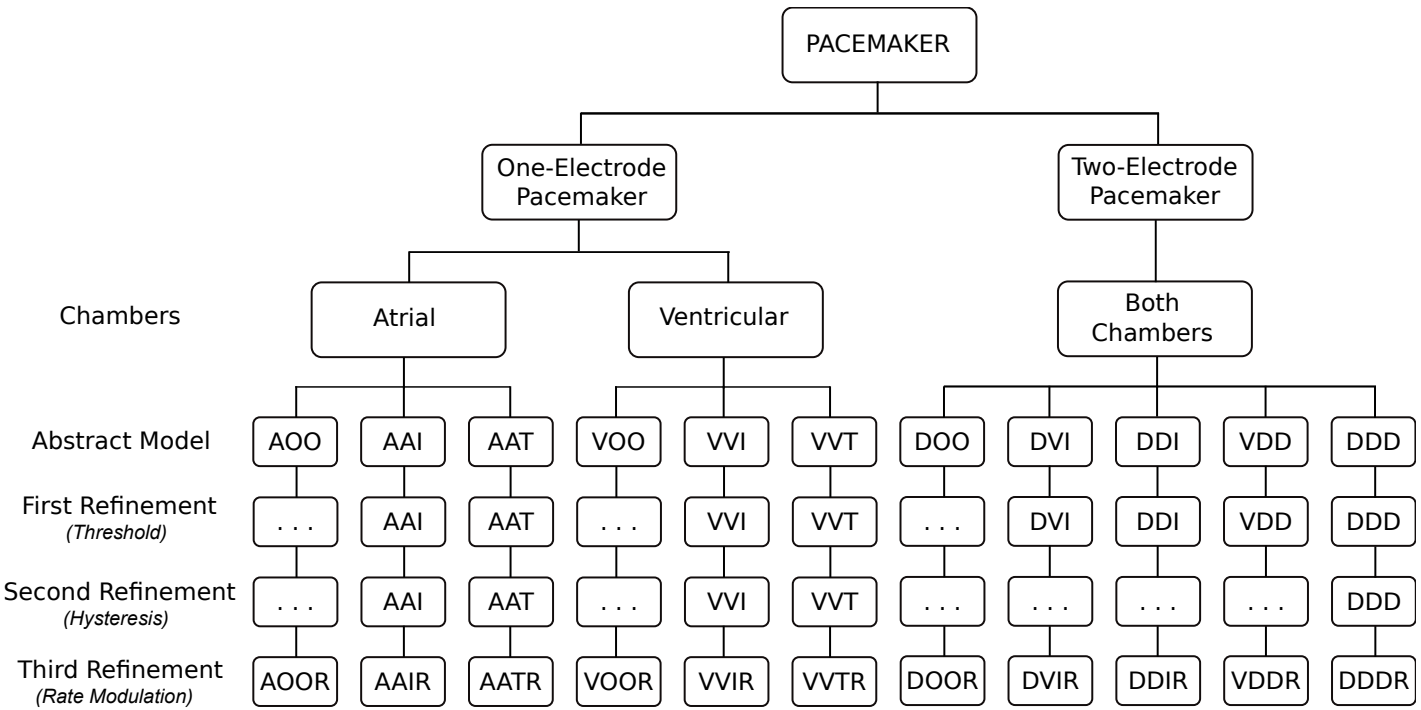
The Cardiac Pacemaker

Design Patterns

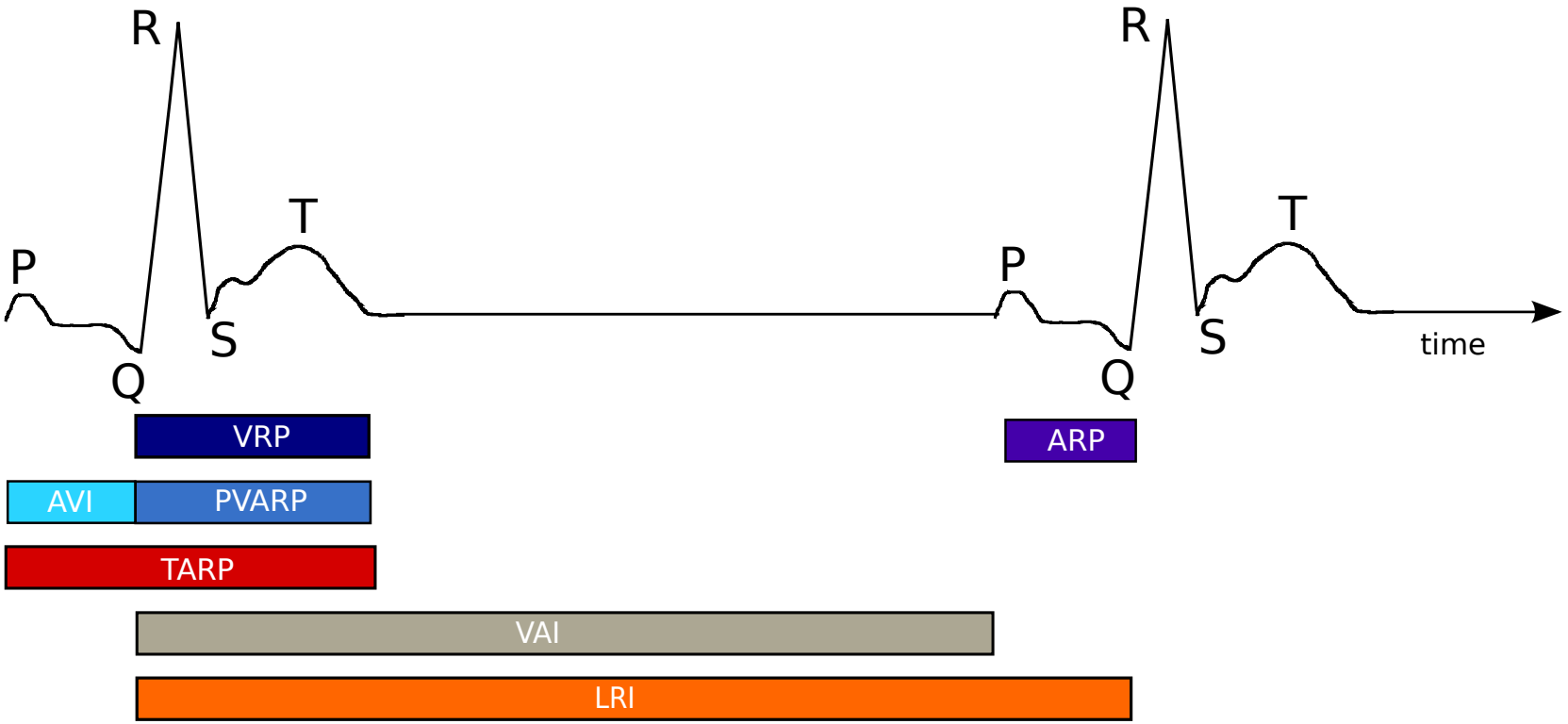
- Action Reaction Patterns
- Real Time Patterns

Development of Cardiac Pacemaker

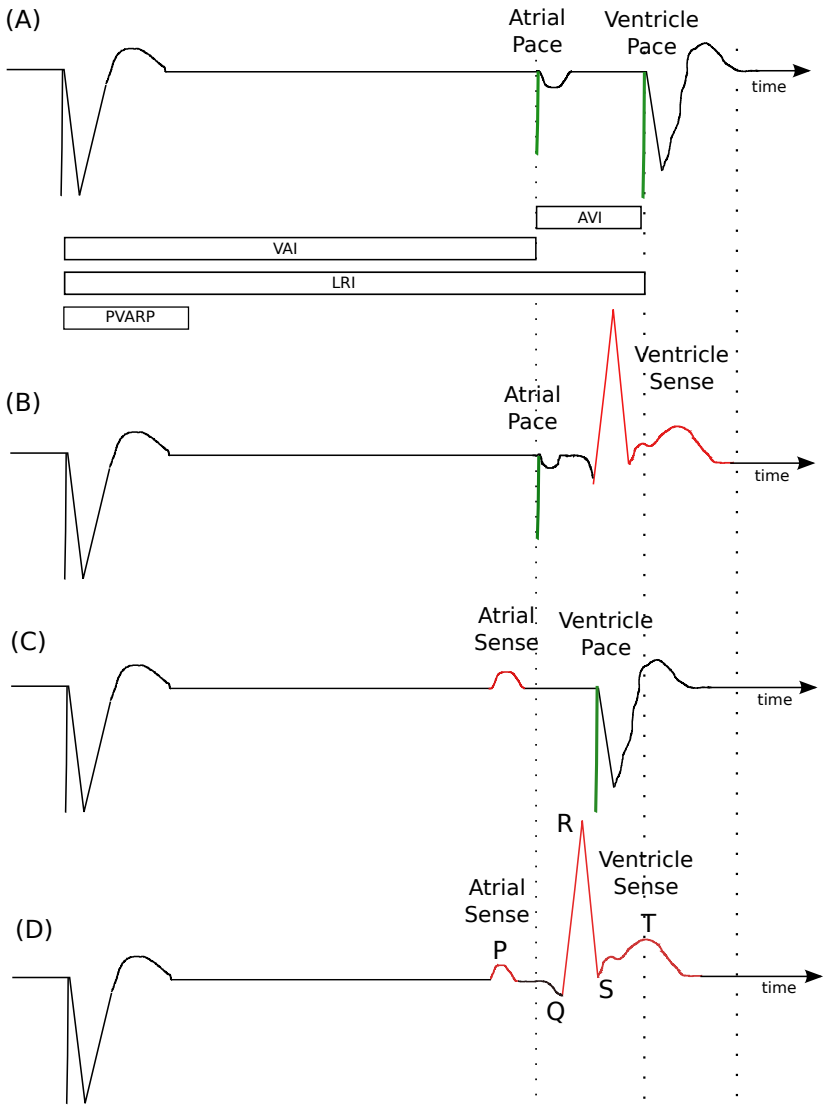
- Formalization of 1 and 2-electrode cardiac pacemaker



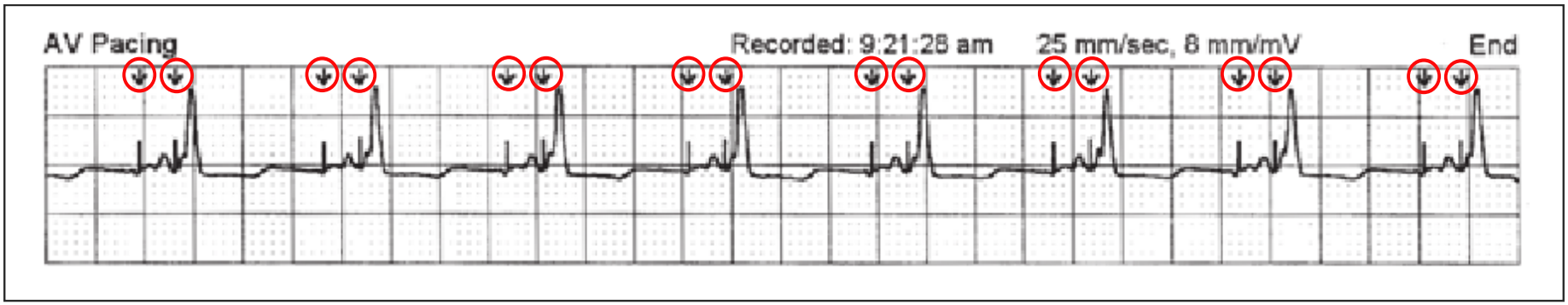
Timing Cycles



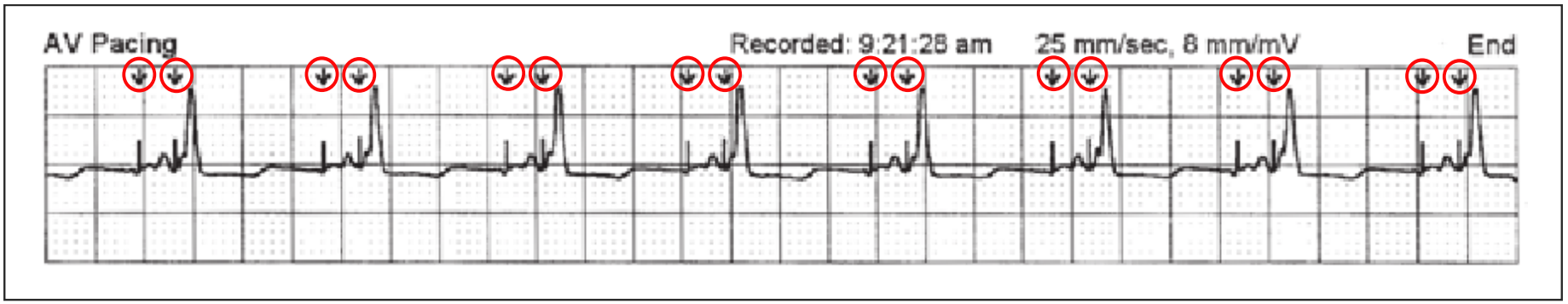
The DDD Pacing Scenarios



DDD Pacing Modes

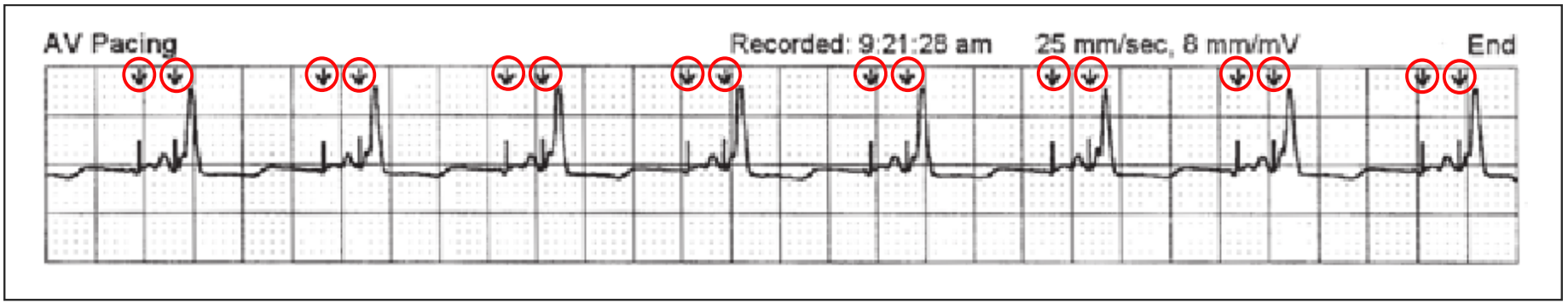


DDD Pacing Modes



$axm1 : LRL \in 30 \mathinner{..} 175$
$axm2 : URL \in 50 \mathinner{..} 175$
$axm3 : URI \in \mathbb{N}_1 \ \wedge URI = 60000/URL$
$axm4 : LRI \in \mathbb{N}_1 \ \wedge LRI = 60000/LRL$
$axm5 : status = \{ON, OFF\}$
$axm6 : FixedAV \in 70 \mathinner{..} 300$
$axm7 : ARP \in 150 \mathinner{..} 500$
$axm8 : VRP \in 150 \mathinner{..} 500$
$axm9 : PVARP \in 150 \mathinner{..} 500$
$axm10 : V\_Blank \in 30 \mathinner{..} 60$
$\ldots$

DDD Pacing Modes



$axm1 : LRL \in 30 \mathinner{..} 175$
$axm2 : URL \in 50 \mathinner{..} 175$
$axm3 : URI \in \mathbb{N}_1 \ \wedge URI = 60000 / URL$
$axm4 : LRI \in \mathbb{N}_1 \ \wedge LRI = 60000 / LRL$
$axm5 : status = \{ON, OFF\}$
$axm6 : FixedAV \in 70 \mathinner{..} 300$
$axm7 : ARP \in 150 \mathinner{..} 500$
$axm8 : VRP \in 150 \mathinner{..} 500$
$axm9 : PVARP \in 150 \mathinner{..} 500$
$axm10 : V\_Blank \in 30 \mathinner{..} 60$
. . .

$inv1 : PM\_Actuator\_A \in status$
$inv2 : PM\_Sensor\_A \in status$
$inv5 : Pace\_Int \in URI \mathinner{..} LRI$
$inv6 : sp \in 1 \mathinner{..} Pace\_Int$
$inv7 : last\_sp \geq PVARP \wedge last\_sp \leq Pace\_Int$

$inv11 : sp < VRP \wedge AV\_Count\_STATE = FALSE \Rightarrow$
$PM\_Actuator\_V = OFF \wedge PM\_Sensor\_A = OFF \wedge$
$PM\_Sensor\_V = OFF \wedge PM\_Actuator\_A = OFF$

$inv12 : Pace\_Int\_flag = FALSE \wedge PM\_Actuator\_V = ON \Rightarrow$
$sp = Pace\_Int \vee (sp < Pace\_Int \wedge$
$AV\_Count > V\_Blank \wedge AV\_Count \geq FixedAV)$

$inv13 : Pace\_Int\_flag = FALSE \wedge PM\_Actuator\_A = ON \Rightarrow$
$(sp \geq Pace\_Int - FixedAV)$
. . .

DDD Pacing Modes

EVENT Actuator_ON_V
WHEN
grd1 : $PM\_Actuator\_V = OFF$
grd2 : $(sp = Pace\_Int)$
$\lor$
$(sp < Pace\_Int \land AV\_Count > V\_Blank \land AV\_Count \geq FixedAV)$
grd3 : $sp \geq VRP \land sp \geq PVARP$
THEN
act1 : $PM\_Actuator\_V := ON$
act2 : $last\_sp := sp$
END

EVENT Actuator_OFF_V
WHEN
grd1 : $PM\_Actuator\_V = ON$
grd2 : $(sp = Pace\_Int)$
$\lor$
$(sp < Pace\_Int \land AV\_Count > V\_Blank \land AV\_Count \geq FixedAV$
grd3 : $AV\_Count\_STATE = TRUE$
grd4 : $PM\_Actuator\_A = OFF$
grd5 : $PM\_Sensor\_A = OFF$
THEN
act1 : $PM\_Actuator\_V := OFF$
act2 : $AV\_Count := 0$
act3 : $AV\_Count\_STATE := FALSE$
act4 : $PM\_Sensor\_V := OFF$
act5 : $sp := 1$
END

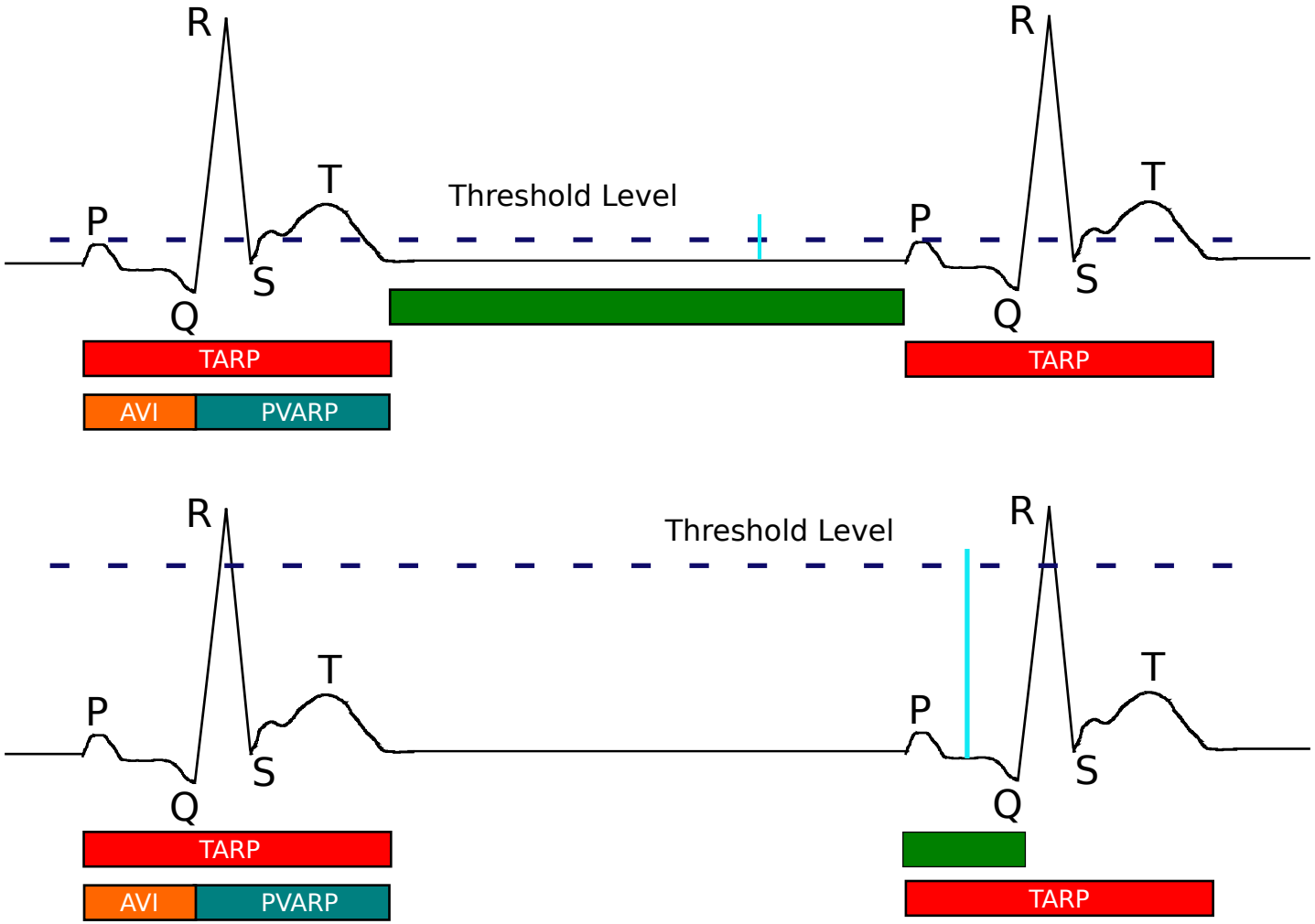
EVENT tic
WHEN
grd1 : $(sp < VRP)$
$\lor$
$(sp \geq VRP \land sp < Pace\_Int - FixedAV \land PM\_Sensor\_A = ON \land PM\_Sensor\_V = ON$
THEN
act1 : $sp := sp + 1$
END

DDD Pacing Modes

EVENT Sensor_ON_V
WHEN
grd1 : $PM\_Sensor\_V = OFF$
grd2 : $(sp \geq VRP \wedge sp < Pace\_Int - FixedAV \wedge PM\_Sensor\_A = ON)$
$\vee$
$(sp \geq Pace\_Int - FixedAV \wedge AV\_Count\_STATE = TRUE)$
grd4 : $PM\_Actuator\_A = OFF$
THEN
act1 : $PM\_Sensor\_V := ON$
END

EVENT Sensor_OFF_V
WHEN
grd1 : $PM\_Sensor\_V = ON$
grd2 : $(sp < Pace\_Int - FixedAV)$
$\vee$
$(sp \geq Pace\_Int - FixedAV \wedge sp < Pace\_Int)$
grd4 : $PM\_Actuator\_V = OFF$
grd5 : $PM\_Actuator\_A = OFF$
grd6 : $sp \geq VRP \wedge sp \geq PVARP$
THEN
act1 : $PM\_Sensor\_V := OFF$
act2 : $last\_sp := sp$
act3 : $sp := 1$
act5 : $PM\_Sensor\_A := OFF$
act6 : $AV\_Count := 0$
act7 : $AV\_Count\_STATE := FALSE$
END

First Refinement (Threshold): Sensor Activity in DDD



First Refinement (Threshold): Sensor Activity in DDD

$inv1 : Thr\_A \in \mathbb{N}_1 \land Thr\_V \in \mathbb{N}_1$

$inv2 : Pace\_Int\_flag = FALSE \land sp > VRP \land sp < Pace\_Int - FixedAV \Rightarrow PM\_Sensor\_V = ON$

$inv3 : Pace\_Int\_flag = FALSE \land sp > Pace\_Int - FixedAV \land sp < Pace\_Int \land AV\_Count\_STATE = TRUE \Rightarrow$
$PM\_Sensor\_A = OFF \land PM\_Sensor\_V = ON \land PM\_Actuator\_A = OFF$

EVENT Thr_Value_A
ANY
Thr_A_val
WHERE
grd1 : $PM\_Sensor\_A = ON$
grd2 : $Thr\_A\_val \in \mathbb{N}$
grd3 : $Thr\_A\_State = TRUE$
grd5 $Thr\_A < STA\_THR\_A$
grd6 $(sp \geq VRP \land sp < Pace\_Int - FixedAV)$
THEN
act1 : $Thr\_A := Thr\_A\_val$
act2 : $Thr\_A\_State := FALSE$
END

Second Refinement: Hysteresis

What is Hysteresis?

EVENT Hyt_Pace_Updating Refines Change_Pace_Int
ANY
Hyt_Pace_Int
WHERE
grd1 : $Pace\_Int\_flag = TRUE$
grd2 : $Hyt\_Pace\_Int\_flag = TRUE$
grd3 : $Hyt\_Pace\_Int \in Pace\_Int \mathinner{..} LRI$
THEN
act1 : $Pace\_Int := Hyt\_Pace\_Int$
act2 : $Hyt\_Pace\_Int\_flag := FALSE$
act3 : $HYT\_State := TRUE$
END

Third Refinement: Rate Modulation

What is Rate Modulation?

Increase_Interval Refines Change_Pace_Int
WHEN
grd1 : $grd1 : Pace\_Int\_flag = TRUE$
grd1 : $acler\_sensed \geq threshold$
grd1 : $HYT\_State = FALSE$
THEN
act1 : $Pace\_Int := 60000/MSR$
act1 : $acler\_sensed\_flag := TRUE$
END

inv3 : $acler\_sensed < acc\_thr \wedge acler\_sensed\_flag = TRUE \Rightarrow Pace\_Int = 60000/LRL$
inv4 : $acler\_sensed \geq acc\_thr \wedge acler\_sensed\_flag = TRUE \Rightarrow Pace\_Int = 60000/MSR$

Validation, Proof Statistics and Code Generation

ProB

Used to check an expected behavior of each operating mode.

Proof Statistics

Model	Total number of POs	Automatic Proof	Interactive Proof
One-electrode pacemaker			
Abstract Model	203	199(98%)	4(2%)
First Refinement	48	44(91%)	4(9%)
Second Refinement	12	8(66%)	4(34%)
Third Refinement	105	99(94%)	6(6%)
Two-electrode pacemaker			
Abstract Model	204	195(95%)	9(5%)
First Refinement	234	223(95%)	11(5%)
Second Refinement	3	3(100%)	0(0%)
Third Refinement	83	74(89%)	9(11%)
Total	892	845(94%)	47(6%)

Code Generation using EB2ALL.