CS 773 Winter 2001

Exercises and Presentations 3

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For each of the following four exercises, do the given IMPS exercise. The IMPS exercises are found under the `IMPS-Help -> Exercises` menu. Send the instructor an IMPS file (i.e., a file with a `.t` extension) that includes:

1. All the def-forms in the exercise file and any additional def-forms that you have added.
2. Proofs for all the def-theorems in the file.
3. Comments in square brackets expressing your experiences in doing the exercise. If you find any mistakes in the exercise, identify them in your comments. Most of your mark will be given on the basis of what your comments are.

To receive a full mark, the file must load without any errors and all theorems should be grounded when the `General -> Load IMPS File` command is applied to it after

```
(set (proof-log-port) (standard-output))
```

is evaluated in the `*tea*` buffer.

Exercise 5

12 pts., due no later than 1-MAR-2001.

IMPS groups-exercise.

Exercise 6

12 pts., due no later than 1-MAR-2001.

IMPS indicators-exercise.

Exercise 7

IMPS calculus-exercise.

12 pts., due no later than 15-MAR-2001.

Exercise 8

12 pts., due no later than 15-MAR-2001.

IMPS primes-exercise.

Exercise 9

5 pts., due no later than 15-FEB-2001.

Use `def-constant` in the IMPS theory `h-o-real-arithmetic` to define the notion of an equivalence relation on the real numbers with binary relations represented by binary predicates. Use `def-constant` again to define a binary predicate on the real numbers such that, for real numbers x and y, the predicate applied to (x, y) is true iff $|x| = |y|$. Prove that this binary predicate represents an equivalence relation.

Exercise 10

10 pts., due no later than 1-MAR-2001.

Starting from scratch, formalize the theory of (second-order) Peano arithmetic *with constants and axioms for addition and multiplication* as an IMPS theory, and then prove:

1. $4 + 8 = 3 * 4$, i.e., $S^4(0) + S^8(0) = S^3(0) * S^4(0)$ where $S^n(0)$ means S is applied to 0 n times.

2. 0 is the additive identity.

Exercise 11

10 pts., due no later than 1-MAR-2001.

Starting from scratch, formalize the theory of (second-order) Peano arithmetic *without constants and axioms for addition and multiplication as defined in class* as an IMPS theory. Use `def-recursive-constant` to define addition and multiplication in this theory, and prove:

1. $3 * (3 + 4) = (3 * 3) + (3 * 4)$.

2. 1 is the multiplicative identity.

Exercise 12

20 pts., due no later than 30-MAR-2001.

Make your own exercise. Choose a basic topic in mathematics approved by the instructor and develop it using both IMPS and some other theorem proving system.

Exercise 13

40 pts., due no later than 6-APR-2001.

Develop the beginning of group theory using a theorem proving system other than IMPS. Start from the definition of a group and end with the theorem that says a quotient of a group by one of its normal subgroups is a group itself. The corresponding development in IMPS is found in the following IMPS theory library files:

```
theories/groups/groups.t
theories/groups/subgroups.t
theories/groups/group-actions.t
theories/groups/cosets.t
theories/groups/normal-groups.t
```

It is very unlikely that you will be able to follow the IMPS development exactly. Send the instructor the set of files that you produce. The files should be commented and should load in the system you chose to use. A partially completed exercise will be given an appropriate partial mark.

Presentation 2

Due no later than 1-MAR-2001.

In a 10-minute presentation, describe a project in which mechanical theorem proving played a major role. A good example is the work of John Harrison at Intel using HOL to verify floating point algorithms.

Presentation 3

Due no later than 6-APR-2001.

In a 10-minute presentation describe your experiences doing Exercise 12 or 13.