CS 773 Winter 2001

00. Preliminaries

Instructor: W. M. Farmer

Revised: 7 January 2001

Instructor

Dr. William M. Farmer

Office: JHE 336

Extension: 27039

E-mail: `wmfarmer@mcmaster.ca`

Web: `http://imps.mcmaster.ca/wmfarmer`

Office hours: M 2:00–4:00, W 9:30–11:30, R 1:30–2:30

Mission

1. Learn what formalized mathematics is and how to use it in the specification and analysis of complex systems.
2. Learn how to express mathematical models in higher-order logic and set theory.
3. Learn how to use interactive theorem proving systems.

Work Plan

- Lectures in class
- Exercises outside of class
  - Most will be done individually
  - Most will require the use of an interactive theorem proving system
- Student presentations in class
  - Should be about 10 minutes long
- No tests or exams

Text and Web Site

- **Required text**: Imre Lakatos, *Proofs and Refutations*, Cambridge University Press, 1976. ISBN 0-521-29038-4.

- **Course Web site**:

  `http://www.cas.mcmaster.ca/~wmfarmer/CS-773-01`

Tentative Schedule

00. Preliminaries

01. What is Mathematics?

02. The Axiomatic Method

03. Mechanized Mathematics Systems

04. Introduction to IMPS

05. Review of Logic

06. What is Missing from First-Order Logic?

Tentative Schedule (cont.)

07. Higher-order Logic

08. Set Theory

09. Axiomatic Theory Development

10. Conjecture Proving

11. Computation in Proving

12. Common Mathematical Structures

Policy Statements

1. Significant study and reading outside of class is required.

2. Regular class attendance is expected.

3. The student is expected to ask questions during class.

4. The student is welcome to discuss exercises with other students, but exercises must be the student's own work.

5. Suggestions on how to improve the course and the instructor's teaching methods are always welcomed.

Grading

Exercise Points	Presentations	Course Grade
110 or more	3 or more	A^+
100–109	3 or more	A
90–99	3 or more	A^-
85–89	2 or more	B^+
80–94	2 or more	B
75–79	2 or more	B^-
70–74	1 or more	C^+
65–69	1 or more	C
60–64	1 or more	C^-
55–59	0 or more	D^+
50–54	0 or more	D
45–49	0 or more	D^-
44 or below	0 or more	F

Note: The exercises will be worth a total of about 150 points.

Basic Questions

1. What is formalized mathematics?
2. Why is formalized mathematics useful?
3. How is formalized mathematics done?

What is Formalized Mathematics?

- **Formalized mathematics** is conventional mathematics that is expressed and developed within a formal logic
  - What is mathematics?
  - What is a formal logic?
- Formalized mathematics emphasizes the "mathematics", while **formal mathematics** emphasizes the "formality"
- Before the invention of the modern computer, formalized mathematics was of theoretical interest only
  - Biggest precomputer development was Whitehead and Russell's **Principia Mathematica** (1910–1913)

Why is Formalized Mathematics Useful?

- Formalized mathematics can be mechanized with the help of logic and the computer
  - Complexity can be better managed
  - Deduction can be machine checked
  - Thus mathematics can be made easier to do
  - Thus results can be more reliable
- Formalized mathematics can be stored in electronic libraries on the Web
  - Mathematics can be stored as dynamic information
  - Mathematical entities can be reused in multiple contexts
- Complex systems, such as software systems, can be represented and analyzed as mathematical models

How is Formalized Mathematics Done?

- Mathematics is organized using the axiomatic method
  - What is the axiomatic method?
- Mathematics is performed with the assistance of a mechanized mathematics system
  - What is a mechanized mathematics system?
- Axiomatic theories are developed and interconnected
  - What are the methods for developing and interconnecting axiomatic theories?
- Conjectures are formally proved
  - What are the methods of formal proof?