

Theorem Proving Systems

- Support “axiomatic mathematics”
 - Mathematics is represented by axiomatic theories
 - Reasoning is performed by proving conjectures
 - Most emphasize proof checking or proof development

03. Mechanized Mathematics Systems

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- Strengths:
 - Based on rigorous logical foundations
 - Support a wide range of mathematics
- Weaknesses:
 - Very difficult to use
 - Poor support for routine computation
 - Abstract theories are emphasized over concrete structures

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What is a Mechanized Mathematics System?

- A **mechanized mathematics system (MMS)** is a computer environment for doing mathematics in which parts of the mathematics process have been mechanized
 - Employs logic and computing technology
 - Intended to support, improve, and automate the mathematics process
- Two major types of MMS:
 1. **Theorem proving systems** such as Coq, EVES, HOL, IMPS, Isabelle, Mizar, Nqthm, Nuprl, Otter, PVS
 2. **Computer algebra systems** such as Axiom, Macsyma, Maple, and Mathematica

Computer Algebra Systems

- Support “algorithmic mathematics”
 - Mathematics is represented by algorithms
 - Reasoning is performed by computation
- Strengths:
 - Perform fast, sophisticated symbolic computations
 - Relatively easy to use
- Weaknesses:
 - Not based on a rigorous logical foundation
 - Poor support for “context guided” computation
 - Concrete structures are emphasized over abstract theories

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Interactive Mathematics Laboratories

Impact of an IML

- An interactive mathematics laboratory (IML) is an MMS that:
 - Is widely accessible
 - Facilitates many kinds of mathematical activity
 - Combines the capabilities of both computer algebra and theorem proving systems
- An IML offers an environment that is:
 - Formal
 - Interactive
 - Mechanized
- IMLs do not exist today, but much of the technology needed to build one does exist

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Obstacles to Building an IML

1. Mathematics library
 - Mathematical knowledge is stored dynamically
 - Includes both axiomatic and algorithmic mathematics
 - Web accessible
2. Reasoning engine
 - Theory development facility
 - Deduction/computation workbench
3. User interface
 - Supports multiple styles of interaction
 - Offers a range of exploratory tools
 - Provides notational freedom

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IML Research at McMaster

- Goals:
 - Develop an integrated framework for axiomatic and algorithmic mathematics
 - Implement a prototype system based on the framework
- Principal investigators:
 - W. Farmer
 - M. v. Mohrenhildt
 - D. Parnas
- Approach: Generalize and merge the key ideas employed in IMPS, Maple, and Axiom