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03. Mechanized Mathematics Systems

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What is a Mechanized Mathematics System?

- A **mechanized mathematics system (MMS)** is a computer environment for doing mathematics in which parts of the mathematics process have been mechanized
  - Employs logic and computing technology
  - Intended to support, improve, and automate the mathematics process
- Two major types of MMSs:
  1. **Theorem proving systems** such as Coq, EVES, HOL, IMPS, Isabelle, Mizar, Nqthm, Nuprl, Otter, PVS
  2. **Computer algebra systems** such as Axiom, Macsyma, Maple, and Mathematica

Theorem Proving Systems

- Support "axiomatic mathematics"
  - Mathematics is represented by axiomatic theories
  - Reasoning is performed by proving conjectures
  - Most emphasize proof checking or proof development
- Strengths:
  - Based on rigorous logical foundations
  - Support a wide range of mathematics
- Weaknesses:
  - Very difficult to use
  - Poor support for routine computation
  - Abstract theories are emphasized over concrete structures

Computer Algebra Systems

- Support "algorithmic mathematics"
  - Mathematics is represented by algorithms
  - Reasoning is performed by computation
- Strengths:
  - Perform fast, sophisticated symbolic computations
  - Relatively easy to use
- Weaknesses:
  - Not based on a rigorous logical foundation
  - Poor support for "context guided" computation
  - Concrete structures are emphasized over abstract theories

Interactive Mathematics Laboratories

- An **interactive mathematics laboratory (IML)** is an MMS that:
  - Is widely accessible
  - Facilitates many kinds of mathematical activity
  - Combines the capabilities of both computer algebra and theorem proving systems
- An IML offers an environment that is:
  - Formal
  - Interactive
  - Mechanized
- IMLs do not exist today, but much of the technology needed to build one does exist

Components of an IML

1. Mathematics library
   - Mathematical knowledge is stored dynamically
   - Includes both axiomatic and algorithmic mathematics
   - Web accessible
2. Reasoning engine
   - Theory development facility
   - Deduction/computation workbench
3. User interface
   - Supports multiple styles of interaction
   - Offers a range of exploratory tools
   - Provides notational freedom

Impact of an IML

- Transform how people learn and practice mathematics
  - People would have greater mathematical reach
  - Students would learn more mathematics by being able to do more mathematics
- Students, engineers, scientists would likely benefit more than mathematicians
- The mathematical competency of society would be raised

Obstacles to Building an IML

1. The development cost is very high
2. The mathematics community is apathetic
3. Very few people have expertise or training in formalized mathematics
4. There is little interaction between the computer algebra and theorem proving fields
5. To be effective, a mathematics library must include many kinds of mathematics and be carefully organized
6. The design of an IML requires sophisticated software engineering
7. Traditional logics are not suited to be the underlying logic of an IML

IML Research at McMaster

- Goals:
  - Develop an integrated framework for axiomatic and algorithmic mathematics
  - Implement a prototype system based on the framework
- Principal investigators:
  - W. Farmer
  - M. v. Mohrenschildt
  - D. Parnas
- Approach: Generalize and merge the key ideas employed in IMPS, Maple, and Axiom