CS 773 Winter 2001

06. What is Missing From First-Order Logic

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Higher-Order Quantification

- Quantification over predicates or functions is not allowed in FOL
- Many statements cannot be expressed directly in FOL
- Some principles, such as induction, cannot be adequately formalized in FOL

Undefined Terms

- An **undefined term** is a term, like x/x where $x = 0$ and $\lim_{x\to\infty} \sin x$, that has no natural denotation
- Undefined terms are a common and unavoidable part of mathematics
- By assumption, every term in FOL denotes something
- Function symbols in FOL always denote total functions
- Consequently, statements involving undefined terms and partial functions are often awkward to express in FOL

Definite Description

- A **definite description** is a term of the form "the unique x that satisfies the property P"
- Definite description is commonly used in mathematics
- Definite descriptions are not provided in FOL
- Definite descriptions are awkward in a system like FOL in which all terms must be defined

Support for Functions

- A function symbol in FOL is not a term so predicate and function symbols cannot be applied to it
- Function symbols in FOL always denote total functions
- Quantification over functions is not allowed in FOL
- Lambda-notation is not provided in FOL

System for Classifying Terms By Value

- A term classification system is very useful for providing some immediate information about the value of a term before the value of term is known itself
- There is no built-in system for classifying terms by value in FOL (as, for example, in a type theory)

Solutions

1. Use a different logic:
   - Second-order logic (SOL)
   - Simple type theory (STT)
   - Another kind of type theory or higher-order logic
   - Partial first-order logic (PFOL)
   - LUTINS (IMPS logic)
2. Use a special theory in FOL:
   - Zermelo-Fraenkel (ZF) set theory
   - Von-Neumann-Bernays-Gödel (NBG) set theory
   - Another kind of set theory or function theory
3. Use a special theory in a different logic:
   - A Set Theory for Mechanized Mathematics (STMM)