CS 773 Winter 2002

Exercise Set 4

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Exercise 7

20 pts., due no later than 22-MAR-2002, optional

Starting from scratch, create an IMPS theory of Peano arithmetic called `peano1` with the constants 0, S, $+$, and $*$ and the following axioms:

(1) 0 is not a successor.

(2) S is injective.

(3) Induction.

(4) The two axioms that specify $+$ recursively.

(5) The two axioms that specify $*$ recursively.

Prove the following theorems in `peano1`:

(1) 0 is the additive identity.

(2) $S(0)$ is the multiplicative identity.

(3) $+$ is associative.

(4) $*$ is associative.

(5) $+$ is commutative.

(6) $*$ is commutative.

Send the instructor your IMPS file with comments and proofs.

Exercise 8

20 pts., due no later than 22-MAR-2002, optional

Create a compound macete that reduces any ground expression of `peano1` (i.e., any expression containing no variables and no constants other than 0, S, $+$, and $*$) to an expression of the form $S^m(0)$ (i.e., S applied to 0 m times).

Give several examples expressed as theorems to show that it works. Send the instructor your IMPS file with comments and proofs.

Exercise 9

20 pts., due no later than 29-MAR-2002, optional

Create the following three interpretations:

(1) An interpretation of `peano1` in `h-o-real-arithmetic`.

(2) An interpretation of `commutative-monoid-theory` in the additive part of `peano1`.

(3) An interpretation of `commutative-monoid-theory` in the multiplicative part of `peano1`.

Send the instructor your IMPS file with comments and proofs.

Exercise 10

20 pts., due no later than 5-APR-2002, optional

Starting from scratch, create an IMPS theory of Peano arithmetic called `peano2` with the constants 0 and S and the following axioms:

(1) 0 is not a successor.

(2) S is injective.

(3) Induction.

Define $+$ and $*$ recursively in `peano2` using `def-recursive-constant`. Do Exercises 7–9 using `peano2` in place of `peano1`. Send the instructor your IMPS file with comments and proofs.