CS 773 Winter 2002

01. What is Formalized Mathematics?

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Basic Questions

1. What is formalized mathematics?
2. Why is formalized mathematics useful?
3. How is formalized mathematics done?

What is Formalized Mathematics?

- **Formalized mathematics** is conventional mathematics that is expressed and developed within a formal logic
  - What is mathematics?
  - What is a formal logic?
- Formalized mathematics emphasizes the "mathematics", while **formal mathematics** emphasizes the "formality"
- Before the invention of the modern computer, formalized mathematics was of theoretical interest only
  - Biggest precomputer development was Whitehead and Russell's **Principia Mathematica** (1910–1913)

Why is Formalized Mathematics Useful?

- Formalized mathematics can be mechanized with the help of logic and the computer
  - Complexity can be better managed
  - Deduction can be machine checked
  - Thus mathematics can be made easier to do
  - Thus results can be more reliable
- Formalized mathematics can be stored in electronic libraries on the Web
  - Mathematics can be stored as dynamic information
  - Mathematical entities can be reused in multiple contexts
- Complex systems, such as software systems, can be represented and analyzed as mathematical models

How is Formalized Mathematics Done?

- Mathematics is performed with the assistance of a mechanized mathematics system
  - What is a mechanized mathematics system?
- Mathematics is organized using the axiomatic method
  - What is the axiomatic method?
- Axiomatic theories are developed and interconnected
  - What are the methods for developing and interconnecting axiomatic theories?
- Conjectures are formally proved
  - What are the techniques of formal proof?