CS 773 Winter 2002

06. Mechanized Mathematics Systems

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What is a Mechanized Mathematics System?

- A **mechanized mathematics system (MMS)** is a computer environment for doing mathematics in which parts of the mathematics process have been mechanized
  - Employs logic and computing technology
  - Intended to support, improve, and automate the mathematics process
- Two major types of MMSs:
  1. **Computer Theorem proving systems** such as Coq, EVES, HOL, IMPS, Isabelle, Mizar, Nqthm, Nuprl, Otter, PVS
  2. **Computer algebra systems** such as Axiom, Macsyma, Maple, and Mathematica

Computer Theorem Proving Systems

- Support "axiomatic mathematics"
  - Mathematics is represented by axiomatic theories
  - Reasoning is performed by proving conjectures
  - Most emphasize proof checking or proof development
- Strengths:
  - Based on rigorous logical foundations
  - Support a wide range of mathematics
- Weaknesses:
  - Very difficult to use
  - Poor support for routine computation
  - Abstract theories are emphasized over concrete structures

Computer Algebra Systems

- Support "algorithmic mathematics"
  - Mathematics is represented by algorithms
  - Reasoning is performed by computation
- Strengths:
  - Perform fast, sophisticated symbolic computations
  - Relatively easy to use
- Weaknesses:
  - Not based on a rigorous logical foundation
  - Poor support for "context guided" computation
  - Concrete structures are emphasized over abstract theories

Interactive Mathematics Laboratories

- An **interactive mathematics laboratory (IML)** is an MMS that:
  - Is widely accessible
  - Facilitates many kinds of mathematical activity
  - Combines the capabilities of both computer theorem proving systems and computer algebra systems
- An IML offers an environment that is:
  - Formal
  - Interactive
  - Mechanized
- IMLs do not exist today, but much of the technology needed to build one does exist

Components of an IML

1. Mathematics library
   - Mathematical knowledge is stored dynamically
   - Includes both axiomatic and algorithmic mathematics
   - Web accessible
2. Reasoning engine
   - Theory development facility
   - Deduction/computation workbench
3. User interface
   - Supports multiple styles of interaction
   - Offers a range of exploratory tools
   - Provides notational freedom

Impact of an IML

- Transform how people learn and practice mathematics
  - People would have greater mathematical reach
  - Students would learn more mathematics by being able to do more mathematics
- Students, engineers, scientists would likely benefit more than mathematicians
- The mathematical competency of society would be raised

Obstacles to Building an IML

1. The development cost is very high
2. The mathematics community is apathetic
3. Very few people have expertise or training in formalized mathematics
4. There is little interaction between the computer theorem proving and computer algebra fields
5. To be effective, a mathematics library must include many kinds of mathematics and be carefully organized
6. The design of an IML requires sophisticated software engineering
7. Traditional logics are not suited to be the underlying logic of an IML

The MathScheme Project

- **Objective**: To develop a new approach to mechanized mathematics in which computer theorem proving and computer algebra are integrated and generalized
- **Members**:
  - Bill Farmer (project leader)
  - Wolfram Kahl
  - Rheda Khedri
  - Mark Lawford
  - Martin v. Mohrenschildt
  - Dave Parnas
- **Web site**: `http://imps.mcmaster.ca/mathscheme/`

Project Goals and Strategy

- **Goals**:
  1. Develop a **formal framework for managing mathematics**
  2. Develop a **microkernel** for a mechanized mathematics system based on the framework
  3. Build an **interactive mathematics laboratory** on top of the microkernel
- **Strategy**: Build on the ideas embodied in:
  - IMPS theorem proving system
  - Axiom computer algebra system

Principal Ideas

- Facilitate the full mathematics process of creating, exploring, and connecting models
- Allow algorithms to be assumed as axioms
- Merge deduction and computation into a single activity
- Use the little theories method to organized mathematics
  - Distribute mathematical reasoning across a network of axiomatic theories
  - Represent a local context of assumptions as an axiomatic theory
- Allow different background logics to be used simultaneously