SE 2A04 Fall 2000

s01. Logic

Instructor: W. M. Farmer

Revised: 15 September 2000

Inductively Defined Sets

- A set S can be **inductively defined** from a set of base elements B and a set of operations O

- Top-down style: A set T is **inductive** with respect to B and O if the following statements hold:
  - $B \subseteq T$.
  - T is closed under each operation in O.

  S is defined to be the smallest inductive set with respect to B and O.

- Bottom-up style: Let $S_0, S_1, \ldots$ be sets defined as follows:
  - $S_0 = B$.
  - S_n is obtained by applying operators in O to members of B at most n times.

  Lemma: $S_0 \subseteq S_1 \subseteq S_2 \subseteq \cdots$
  S is defined to be $\bigcup_{n=0}^{\infty} S_n$.