Why Mathematical Logic is Useful To Software Developers

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What is Mathematical Logic?

- Study of **mathematical reasoning**
- Branch of mathematics
  - Arose in the late 1800s
- Principal tools: formal systems called **logics**

What is a Logic?

- Model of the **mathematics process**:
  - Formulating mathematical models
  - Exploring these models by stating and proving conjectures and by performing calculations

- Provides:
  - **Formal languages** for modeling
  - Notion of **logical consequence** for proving conjectures

- Logicians study logics to better understand mathematics, but logics can also be used to do mathematics

- Examples: propositional logic, equational logic, first-order predicate logic, simple type theory

Replay of the Development of Calculus

- Logic: IMPS logic
  Underlying theory: theory of real arithmetic

- Definitions, lemmas, theorems, proofs, ...

- $\mathrm{lim} : (\mathbf{R} \to \mathbf{R}) \times \mathbf{R} \to \mathbf{R}$, which gives the limit of a real-valued function at a point, is defined by

$$
\begin{aligned}
\mathrm{lim} \; = \; & \lambda\, f : \mathbf{R} \to \mathbf{R}, a : \mathbf{R} \,.\, \mathrm{I}\, y : \mathbf{R} \,. \\
& \forall\, \epsilon : \mathbf{R} \,.\, \epsilon > 0 \supset \exists\, \delta : \mathbf{R} \,.\, \delta > 0 \supset \\
& \forall\, x : \mathbf{R} \,.\, |x - a| < \delta \wedge x \neq a \supset |f(x) - y| < \epsilon
\end{aligned}
$$

- More definitions, lemmas, theorems, proofs, ...

Benefits of Using a Logic In Software Development

- Improves the mathematical process
  - Underlying theories are made explicit
  - Concepts and statements are expressed precisely
  - Proofs are expressed as formal deductions

- Produces a precise form of documentation

- Enables tools such as theorem provers to be employed in the development process