

# LaTeX for Alonzo

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## Abstract

This document illustrates how to use a set of LaTeX macros and environments for presenting types, expressions, and modules of Alonzo, a practice-oriented version of simple type theory.

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## 1 Introduction

*Alonzo* [1] is a practice-oriented logic based on Alonzo Church’s formulation of *simple type theory* known as *Church’s type theory*. The LaTeX source file

`alonzo-notation.tex`,

whose contents are given in Section 4, contains a set of LaTeX macros for presenting Alonzo types and expressions in both formal and compact notations. It also contains a set of LaTeX environments for presenting Alonzo mathematical knowledge modules. This document illustrates how to use these macros and environments with a series of examples.

## 2 Macros

In this section, each LaTeX macro in

`alonzo-notation.tex`

is presented with metavariable arguments followed by the output it produces.

## 2.1 Macros for Presenting Types in Formal Notation

1. `\fBoolTy`  
BoolTy
2. `\fBaseTy {\mathbf{a}}`  
BaseTy(a)
3. `\fFunTy {\alpha} {\beta}`  
FunTy( $\alpha, \beta$ )
4. `\fProdTy {\alpha} {\beta}`  
ProdTy( $\alpha, \beta$ )

## 2.2 Macros for Presenting Expressions in Formal Notation

1. `\fVar {\mathbf{x}} {\alpha}`  
Var(x,  $\alpha$ )
2. `\fCon {\mathbf{c}} {\alpha}`  
Con(c,  $\alpha$ )
3. `\fEq {\mathbf{A}}_{\alpha} {\mathbf{B}}_{\alpha}`  
Eq( $\mathbf{A}_{\alpha}, \mathbf{B}_{\alpha}$ )
4. `\fFunApp`  
    `{\mathbf{F}}_{\cFunTyX {\alpha} {\beta}}`  
    `{\mathbf{A}}_{\alpha}`  
FunApp( $\mathbf{F}_{\alpha \rightarrow \beta}, \mathbf{A}_{\alpha}$ )
5. `\fFunAbs {\mathbf{x}} {\alpha} {\mathbf{B}}_{\beta}`  
FunAbs(Var(x,  $\alpha$ ),  $\mathbf{B}_{\beta}$ )
6. `\fDefDes {\mathbf{x}} {\alpha} {\mathbf{A}}_{\cB}`  
DefDes(Var(x,  $\alpha$ ),  $\mathbf{A}_o$ )
7. `\fOrdPair {\mathbf{A}}_{\alpha} {\mathbf{B}}_{\beta}`  
OrdPair( $\mathbf{A}_{\alpha}, \mathbf{B}_{\beta}$ )

### 2.3 Macros for Presenting Types in Compact Notation

1. `\cBoolTy` or `\cB`  
 $o$
2. `\cBaseTy` `{\mathbf{a}}`  
 $\mathbf{a}$
3. `\cFunTy` `{\alpha}` `{\beta}`  
 $(\alpha \rightarrow \beta)$
4. `\cFunTyX` `{\alpha}` `{\beta}`  
 $\alpha \rightarrow \beta$
5. `\cFunTyB` `{\alpha}` `{\beta}` `{\gamma}`  
 $(\alpha \rightarrow \beta \rightarrow \gamma)$
6. `\cFunTyBX` `{\alpha}` `{\beta}` `{\gamma}`  
 $\alpha \rightarrow \beta \rightarrow \gamma$
7. `\cFunTyC` `{\alpha}` `{\beta}` `{\gamma}` `{\delta}`  
 $(\alpha \rightarrow \beta \rightarrow \gamma \rightarrow \delta)$
8. `\cFunTyCX` `{\alpha}` `{\beta}` `{\gamma}` `{\delta}`  
 $\alpha \rightarrow \beta \rightarrow \gamma \rightarrow \delta$
9. `\cProdTy` `{\alpha}` `{\beta}`  
 $(\alpha \times \beta)$
10. `\cProdTyX` `{\alpha}` `{\beta}`  
 $\alpha \times \beta$
11. `\cProdTyB` `{\alpha}` `{\beta}` `{\gamma}`  
 $(\alpha \times \beta \times \gamma)$
12. `\cProdTyBX` `{\alpha}` `{\beta}` `{\gamma}`  
 $\alpha \times \beta \times \gamma$
13. `\cProdTyC` `{\alpha}` `{\beta}` `{\gamma}` `{\delta}`  
 $(\alpha \times \beta \times \gamma \times \delta)$

14. `\cProdTyCX`  $\{\alpha\}$   $\{\beta\}$   $\{\gamma\}$   $\{\delta\}$   
 $\alpha \times \beta \times \gamma \times \delta$

## 2.4 Macros for Presenting Expressions in Compact Notation

1. `\cVar`  $\{\mathbf{x}\}$   $\{\alpha\}$   
 $(\mathbf{x} : \alpha)$
2. `\cVarY`  $\{\mathbf{x}\}$   $\{\alpha\}$   
 $\mathbf{x}$
3. `\cCon`  $\{\mathbf{c}\}$   $\{\alpha\}$   
 $\mathbf{c}_\alpha$
4. `\cConY`  $\{\mathbf{c}\}$   $\{\alpha\}$   
 $\mathbf{c}$
5. `\cEq`  $\{\mathbf{A}\}_\alpha$   $\{\mathbf{B}\}_\alpha$   
 $(\mathbf{A}_\alpha = \mathbf{B}_\alpha)$
6. `\cEqX`  $\{\mathbf{A}\}_\alpha$   $\{\mathbf{B}\}_\alpha$   
 $\mathbf{A}_\alpha = \mathbf{B}_\alpha$
7. `\cFunApp`  
 $\{\mathbf{F}\}_\alpha$   $\{\mathbf{A}\}_\alpha$   
 $(\mathbf{F}_{\alpha \rightarrow \beta} \mathbf{A}_\alpha)$
8. `\cFunAppX`  
 $\{\mathbf{F}\}_\alpha$   $\{\mathbf{A}\}_\alpha$   
 $\mathbf{F}_{\alpha \rightarrow \beta} \mathbf{A}_\alpha$
9. `\cFunAppB`  
 $\{\mathbf{F}\}_\alpha$   $\{\mathbf{A}\}_\alpha$   $\{\mathbf{B}\}_\beta$   
 $(\mathbf{F}_{\alpha \rightarrow \beta \rightarrow \gamma} \mathbf{A}_\alpha \mathbf{B}_\beta)$

10. `\cFunAppBX`  

$$\{\mathbf{F}\}_{\{\cFunTyBX \ \{\alpha\} \ \{\beta\} \ \{\gamma\}\}}$$

$$\{\mathbf{A}\}_{\alpha}$$

$$\{\mathbf{B}\}_{\beta}$$

$$\mathbf{F}_{\alpha \rightarrow \beta \rightarrow \gamma} \mathbf{A}_{\alpha} \mathbf{B}_{\beta}$$
11. `\cFunAppC`  

$$\{\mathbf{F}\}_{\{\cFunTyCX \ \{\alpha\} \ \{\beta\} \ \{\gamma\} \ \{\delta\}\}}$$

$$\{\mathbf{A}\}_{\alpha}$$

$$\{\mathbf{B}\}_{\beta}$$

$$\{\mathbf{C}\}_{\gamma}$$

$$(\mathbf{F}_{\alpha \rightarrow \beta \rightarrow \gamma \rightarrow \delta} \mathbf{A}_{\alpha} \mathbf{B}_{\beta} \mathbf{C}_{\gamma})$$
12. `\cFunAppCX`  

$$\{\mathbf{F}\}_{\{\cFunTyCX \ \{\alpha\} \ \{\beta\} \ \{\gamma\} \ \{\delta\}\}}$$

$$\{\mathbf{A}\}_{\alpha}$$

$$\{\mathbf{B}\}_{\beta}$$

$$\{\mathbf{C}\}_{\gamma}$$

$$\mathbf{F}_{\alpha \rightarrow \beta \rightarrow \gamma \rightarrow \delta} \mathbf{A}_{\alpha} \mathbf{B}_{\beta} \mathbf{C}_{\gamma}$$
13. `\cFunAbs`  $\{\mathbf{x}\}$   $\{\alpha\}$   $\{\mathbf{B}\}_{\beta}$   

$$(\lambda \mathbf{x} : \alpha . \mathbf{B}_{\beta})$$
14. `\cFunAbsX`  $\{\mathbf{x}\}$   $\{\alpha\}$   $\{\mathbf{B}\}_{\beta}$   

$$\lambda \mathbf{x} : \alpha . \mathbf{B}_{\beta}$$
15. `\cDefDes`  $\{\mathbf{x}\}$   $\{\alpha\}$   $\{\mathbf{A}\}_{\{\cB\}}$   

$$(\mathbf{I} \mathbf{x} : \alpha . \mathbf{A}_o)$$
16. `\cDefDesX`  $\{\mathbf{x}\}$   $\{\alpha\}$   $\{\mathbf{A}\}_{\{\cB\}}$   

$$\mathbf{I} \mathbf{x} : \alpha . \mathbf{A}_o$$

17. `\cOrdPair`  $\{\mathbf{A}_\alpha\}$   $\{\mathbf{B}_\beta\}$   
 $(\mathbf{A}_\alpha, \mathbf{B}_\beta)$

## 2.5 Macros for Boolean Operators

1. `\cTPC` or `\cT`  
 $T_o$
2. `\cFPC` or `\cF`  
 $F_o$
3. `\cAndPC`  
 $\wedge_{o \rightarrow o \rightarrow o}$
4. `\cAnd`  $\{\mathbf{A}_\cB\}$   $\{\mathbf{B}_\cB\}$   
 $(\mathbf{A}_o \wedge \mathbf{B}_o)$
5. `\cAndX`  $\{\mathbf{A}_\cB\}$   $\{\mathbf{B}_\cB\}$   
 $\mathbf{A}_o \wedge \mathbf{B}_o$
6. `\cAndB`  
 $\{\mathbf{A}_\cB\}$   
 $\{\mathbf{B}_\cB\}$   
 $\{\mathbf{C}_\cB\}$   
 $(\mathbf{A}_o \wedge \mathbf{B}_o \wedge \mathbf{C}_o)$
7. `\cAndBX`  
 $\{\mathbf{A}_\cB\}$   
 $\{\mathbf{B}_\cB\}$   
 $\{\mathbf{C}_\cB\}$   
 $\mathbf{A}_o \wedge \mathbf{B}_o \wedge \mathbf{C}_o$
8. `\cAndL`  
 $\{\mathbf{A}^{\c1}_\cB\}$   
 $\backslash\text{And}$   
 $\backslash\text{cdots}$   
 $\backslash\text{And}$   
 $\{\mathbf{A}^{\c{n}}_\cB\}$   
 $(\mathbf{A}_o^1 \ \& \ \dots \ \& \ \mathbf{A}_o^n)$

9. `\cAndLX`  

$$\{\mathbf{A}^{\wedge\{1\}}_{\cB}$$

$$\And$$

$$\cdots$$

$$\And$$

$$\mathbf{A}^{\wedge\{n\}}_{\cB}$$

$$\mathbf{A}_o^1 \& \cdots \& \mathbf{A}_o^n$$
10. `\cImpliesPC`  

$$\Rightarrow_{o \rightarrow o \rightarrow o}$$
11. `\cImplies`  $\{\mathbf{A}_{\cB}\}$   $\{\mathbf{B}_{\cB}\}$   

$$(\mathbf{A}_o \Rightarrow \mathbf{B}_o)$$
12. `\cImpliesX`  $\{\mathbf{A}_{\cB}\}$   $\{\mathbf{B}_{\cB}\}$   

$$\mathbf{A}_o \Rightarrow \mathbf{B}_o$$
13. `\cNegPC`  

$$\neg_{o \rightarrow o}$$
14. `\cNeg`  $\{\mathbf{A}_{\cB}\}$   

$$(\neg \mathbf{A}_o)$$
15. `\cNegX`  $\{\mathbf{A}_{\cB}\}$   

$$\neg \mathbf{A}_o$$
16. `\cOrPC`  

$$\vee_{o \rightarrow o \rightarrow o}$$
17. `\cOr`  $\{\mathbf{A}_{\cB}\}$   $\{\mathbf{B}_{\cB}\}$   

$$(\mathbf{A}_o \vee \mathbf{B}_o)$$
18. `\cOrX`  $\{\mathbf{A}_{\cB}\}$   $\{\mathbf{B}_{\cB}\}$   

$$\mathbf{A}_o \vee \mathbf{B}_o$$
19. `\cOrB`  

$$\{\mathbf{A}_{\cB}\}$$

$$\{\mathbf{B}_{\cB}\}$$

$$\{\mathbf{C}_{\cB}\}$$

$$(\mathbf{A}_o \vee \mathbf{B}_o \vee \mathbf{C}_o)$$



20. `\cOrBX`  

$$\{\mathbf{A}\}_{\mathbf{cB}}$$

$$\{\mathbf{B}\}_{\mathbf{cB}}$$

$$\{\mathbf{C}\}_{\mathbf{cB}}$$

$$\mathbf{A}_o \vee \mathbf{B}_o \vee \mathbf{C}_o$$
21. `\cOrL`  

$$\{\mathbf{A}^{\{1\}}\}_{\mathbf{cB}}$$

$$\backslash\text{Or}$$

$$\backslash\text{cdots}$$

$$\backslash\text{Or}$$

$$\{\mathbf{A}^{\{n\}}\}_{\mathbf{cB}}$$

$$(\mathbf{A}_o^1 \vee \dots \vee \mathbf{A}_o^n)$$
22. `\cOrLX`  

$$\{\mathbf{A}^{\{1\}}\}_{\mathbf{cB}}$$

$$\backslash\text{Or}$$

$$\backslash\text{cdots}$$

$$\backslash\text{Or}$$

$$\{\mathbf{A}^{\{n\}}\}_{\mathbf{cB}}$$

$$\mathbf{A}_o^1 \vee \dots \vee \mathbf{A}_o^n$$

## 2.6 Macros for Binary Operators

1. `\cBin`  

$$\{\mathbf{A}\}_{\alpha}$$

$$\{\mathbf{c}\}$$

$$\{\alpha\} \{\alpha\} \{\beta\}$$

$$\{\mathbf{B}\}_{\alpha}$$

$$(\mathbf{A}_\alpha \mathbf{c}_{\alpha \rightarrow \alpha \rightarrow \beta} \mathbf{B}_\alpha)$$
2. `\cBinX`  

$$\{\mathbf{A}\}_{\alpha}$$

$$\{\mathbf{c}\}$$

$$\{\alpha\} \{\alpha\} \{\beta\}$$

$$\{\mathbf{B}\}_{\alpha}$$

$$\mathbf{A}_\alpha \mathbf{c}_{\alpha \rightarrow \alpha \rightarrow \beta} \mathbf{B}_\alpha$$

3. `\cBinB`

$$\{\mathbf{A}\}_{\alpha}$$

$$\{\cCon$$

$$\{\mathbf{c}\}$$

$$\{\cFunTyBX \{\alpha\} \{\alpha\} \{\beta\}\}$$

$$\{\mathbf{B}\}_{\alpha}$$

$$\{\cCon$$

$$\{\mathbf{d}\}$$

$$\{\cFunTyBX \{\alpha\} \{\alpha\} \{\beta\}\}$$

$$\{\mathbf{C}\}_{\alpha}$$

$$(\mathbf{A}_{\alpha} \mathbf{c}_{\alpha \rightarrow \alpha \rightarrow \beta} \mathbf{B}_{\alpha} \mathbf{d}_{\alpha \rightarrow \alpha \rightarrow \beta} \mathbf{C}_{\alpha})$$
4. `\cBinBX`

$$\{\mathbf{A}\}_{\alpha}$$

$$\{\cCon$$

$$\{\mathbf{c}\}$$

$$\{\cFunTyBX \{\alpha\} \{\alpha\} \{\beta\}\}$$

$$\{\mathbf{B}\}_{\alpha}$$

$$\{\cCon$$

$$\{\mathbf{d}\}$$

$$\{\cFunTyBX \{\alpha\} \{\alpha\} \{\beta\}\}$$

$$\{\mathbf{C}\}_{\alpha}$$

$$\mathbf{A}_{\alpha} \mathbf{c}_{\alpha \rightarrow \alpha \rightarrow \beta} \mathbf{B}_{\alpha} \mathbf{d}_{\alpha \rightarrow \alpha \rightarrow \beta} \mathbf{C}_{\alpha}$$
5. `\cIff`  $\{\mathbf{A}\}_{\cB}$   $\{\mathbf{B}\}_{\cB}$ 

$$(\mathbf{A}_o \Leftrightarrow \mathbf{B}_o)$$
6. `\cIffX`  $\{\mathbf{A}\}_{\cB}$   $\{\mathbf{B}\}_{\cB}$ 

$$\mathbf{A}_o \Leftrightarrow \mathbf{B}_o$$
7. `\cNotEq`  $\{\mathbf{A}\}_{\alpha}$   $\{\mathbf{B}\}_{\alpha}$ 

$$(\mathbf{A}_{\alpha} \neq \mathbf{B}_{\alpha})$$
8. `\cNotEqX`  $\{\mathbf{A}\}_{\alpha}$   $\{\mathbf{B}\}_{\alpha}$ 

$$\mathbf{A}_{\alpha} \neq \mathbf{B}_{\alpha}$$

## 2.7 Macros for Quantifiers

1. `\cForall`  $\{\mathbf{x}\}$   $\{\alpha\}$   $\{\mathbf{A}\}_{\cB}$ 

$$(\forall \mathbf{x} : \alpha . \mathbf{A}_o)$$

2. `\cforallX {\mathbf{x}} {\alpha} {\mathbf{A}_\cB}`

$\forall \mathbf{x} : \alpha . \mathbf{A}_o$

3. `\cforallB`

`{\mathbf{x}}`

`{\alpha}`

`{\mathbf{y}}`

`{\beta}`

`{\mathbf{A}_\cB}`

$(\forall \mathbf{x} : \alpha, \mathbf{y} : \beta . \mathbf{A}_o)$

4. `\cforallBX`

`{\mathbf{x}}`

`{\alpha}`

`{\mathbf{y}}`

`{\beta}`

`{\mathbf{A}_\cB}`

$\forall \mathbf{x} : \alpha, \mathbf{y} : \beta . \mathbf{A}_o$

5. `\cforallC`

`{\mathbf{x}}`

`{\alpha}`

`{\mathbf{y}}`

`{\beta}`

`{\mathbf{z}}`

`{\gamma}`

`{\mathbf{A}_\cB}`

$(\forall \mathbf{x} : \alpha, \mathbf{y} : \beta, \mathbf{z} : \gamma . \mathbf{A}_o)$

6. `\cforallCX`

`{\mathbf{x}}`

`{\alpha}`

`{\mathbf{y}}`

`{\beta}`

`{\mathbf{z}}`

`{\gamma}`

`{\mathbf{A}_\cB}`

$\forall \mathbf{x} : \alpha, \mathbf{y} : \beta, \mathbf{z} : \gamma . \mathbf{A}_o$

7.  $\text{\cForsome}\{\mathbf{x}\}\{\alpha\}\{\mathbf{A}_\text{cB}\}$   
 $(\exists x : \alpha . \mathbf{A}_o)$
8.  $\text{\cForsomeX}\{\mathbf{x}\}\{\alpha\}\{\mathbf{A}_\text{cB}\}$   
 $\exists x : \alpha . \mathbf{A}_o$
9.  $\text{\cForsomeB}$   
 $\{\mathbf{x}\}$   
 $\{\alpha\}$   
 $\{\mathbf{y}\}$   
 $\{\beta\}$   
 $\{\mathbf{A}_\text{cB}\}$   
 $(\exists x : \alpha, y : \beta . \mathbf{A}_o)$
10.  $\text{\cForsomeBX}$   
 $\{\mathbf{x}\}$   
 $\{\alpha\}$   
 $\{\mathbf{y}\}$   
 $\{\beta\}$   
 $\{\mathbf{A}_\text{cB}\}$   
 $\exists x : \alpha, y : \beta . \mathbf{A}_o$
11.  $\text{\cForsomeC}$   
 $\{\mathbf{x}\}$   
 $\{\alpha\}$   
 $\{\mathbf{y}\}$   
 $\{\beta\}$   
 $\{\mathbf{z}\}$   
 $\{\gamma\}$   
 $\{\mathbf{A}_\text{cB}\}$   
 $(\exists x : \alpha, y : \beta, z : \gamma . \mathbf{A}_o)$
12.  $\text{\cForsomeCX}$   
 $\{\mathbf{x}\}$   
 $\{\alpha\}$   
 $\{\mathbf{y}\}$   
 $\{\beta\}$   
 $\{\mathbf{z}\}$   
 $\{\gamma\}$

$\{\mathbf{A}\}_{\mathbf{B}}$

$\exists \mathbf{x} : \alpha, \mathbf{y} : \beta, \mathbf{z} : \gamma . \mathbf{A}_o$

13.  $\backslash\text{cForsomeUnique}$   
 $\{\mathbf{x}\}$   
 $\{\alpha\}$   
 $\{\mathbf{A}\}_{\mathbf{B}}$

$(\exists! \mathbf{x} : \alpha . \mathbf{A}_o)$

14.  $\backslash\text{cForsomeUniqueX}$   
 $\{\mathbf{x}\}$   
 $\{\alpha\}$   
 $\{\mathbf{A}\}_{\mathbf{B}}$

$\exists! \mathbf{x} : \alpha . \mathbf{A}_o$

## 2.8 Macros for Definedness

1.  $\backslash\text{cBotPC } \{\alpha\}$

$\perp_\alpha$

2.  $\backslash\text{cEmpFunPC } \{\alpha\} \{\beta\}$

$\Delta_{\alpha \rightarrow \beta}$

3.  $\backslash\text{cIsDef } \{\mathbf{A}\}_{\alpha}$

$(\mathbf{A}_\alpha \downarrow)$

4.  $\backslash\text{cIsDefX } \{\mathbf{A}\}_{\alpha}$

$\mathbf{A}_\alpha \downarrow$

5.  $\backslash\text{cIsUndef } \{\mathbf{A}\}_{\alpha}$

$(\mathbf{A}_\alpha \uparrow)$

6.  $\backslash\text{cIsUndefX } \{\mathbf{A}\}_{\alpha}$

$\mathbf{A}_\alpha \uparrow$

7.  $\backslash\text{cQuasiEq } \{\mathbf{A}\}_{\alpha} \{\mathbf{B}\}_{\alpha}$

$(\mathbf{A}_\alpha \simeq \mathbf{B}_\alpha)$

8.  $\backslash\text{cQuasiEqX } \{\mathbf{A}\}_{\alpha} \{\mathbf{B}\}_{\alpha}$

$\mathbf{A}_\alpha \simeq \mathbf{B}_\alpha$

9. `\cNotQuasiEq`  $\{\mathbf{A}\}_{\alpha}$   $\{\mathbf{B}\}_{\alpha}$   
 $(\mathbf{A}_{\alpha} \not\approx \mathbf{B}_{\alpha})$
10. `\cNotQuasiEqX`  $\{\mathbf{A}\}_{\alpha}$   $\{\mathbf{B}\}_{\alpha}$   
 $\mathbf{A}_{\alpha} \not\approx \mathbf{B}_{\alpha}$
11. `\cIfThenElse`  
 $\{\mathbf{A}\}_{\mathbf{B}}$   
 $\{\mathbf{B}\}_{\alpha}$   
 $\{\mathbf{C}\}_{\alpha}$   
 $\text{IF}(\mathbf{A}_{\alpha}, \mathbf{B}_{\alpha}, \mathbf{C}_{\alpha})$
12. `\cIf`  
 $\{\mathbf{A}\}_{\mathbf{B}}$   
 $\{\mathbf{B}\}_{\alpha}$   
 $\{\mathbf{C}\}_{\alpha}$   
 $(\mathbf{A}_{\alpha} \mapsto \mathbf{B}_{\alpha} \mid \mathbf{C}_{\alpha})$
13. `\cIfX`  
 $\{\mathbf{A}\}_{\mathbf{B}}$   
 $\{\mathbf{B}\}_{\alpha}$   
 $\{\mathbf{C}\}_{\alpha}$   
 $\mathbf{A}_{\alpha} \mapsto \mathbf{B}_{\alpha} \mid \mathbf{C}_{\alpha}$

## 2.9 Macros for Sets

1. `\cSetTy`  $\{\alpha\}$   
 $\{\alpha\}$
2. `\cIn`  
 $\{\mathbf{A}\}_{\alpha}$   
 $\{\mathbf{B}\}_{\{\cSetTy \{\alpha\}\}}$   
 $(\mathbf{A}_{\alpha} \in \mathbf{B}_{\{\alpha\}})$
3. `\cInX`  
 $\{\mathbf{A}\}_{\alpha}$   
 $\{\mathbf{B}\}_{\{\cSetTy \{\alpha\}\}}$   
 $\mathbf{A}_{\alpha} \in \mathbf{B}_{\{\alpha\}}$

4. `\cNotIn`  

$$\{\mathbf{A}_{\alpha}\}$$

$$\{\mathbf{B}_{\alpha}\}$$

$$(\mathbf{A}_{\alpha} \notin \mathbf{B}_{\alpha})$$
5. `\cNotInX`  

$$\{\mathbf{A}_{\alpha}\}$$

$$\{\mathbf{B}_{\alpha}\}$$

$$\mathbf{A}_{\alpha} \notin \mathbf{B}_{\alpha}$$
6. `\cSet`  $\{\mathbf{x}\}$   $\{\alpha\}$   $\{\mathbf{A}_{\alpha}\}$   

$$\{\mathbf{x} : \alpha \mid \mathbf{A}_{\alpha}\}$$
7. `\cEmpSetPC`  $\{\alpha\}$   

$$\emptyset_{\alpha}$$
8. `\cEmpSetAltPC`  $\{\alpha\}$   

$$\{\}_{\alpha}$$
9. `\cUnivSetPC`  $\{\alpha\}$   

$$U_{\alpha}$$
10. `\cFinSet`  $\{n\}$   $\{\alpha\}$   

$$n\text{-}\alpha\text{-SET}$$
11. `\cFinSetL`  

$$\{\mathbf{A}^1_{\alpha}\}$$

$$\dots$$

$$\{\mathbf{A}^n_{\alpha}\}$$

$$\{\mathbf{A}^1_{\alpha}, \dots, \mathbf{A}^n_{\alpha}\}$$
12. `\cSubseteqPC`  $\{\alpha\}$   

$$\subseteq_{\alpha \rightarrow \alpha \rightarrow o}$$
13. `\cSubseteq`  

$$\{\mathbf{A}_{\alpha}\}$$

$$\{\mathbf{B}_{\alpha}\}$$

$$(\mathbf{A}_{\alpha} \subseteq \mathbf{B}_{\alpha})$$

14. `\cSubseteqX`  

$$\{\mathbf{A}_{\{\alpha\}}\}_{\{\alpha\}} \subseteq \{\mathbf{B}_{\{\alpha\}}\}_{\{\alpha\}}$$
15. `\cUnionPC`  $\{\alpha\}$   

$$\bigcup_{\{\alpha\} \rightarrow \{\alpha\} \rightarrow \{\alpha\}}$$
16. `\cUnion`  

$$\{\mathbf{A}_{\{\alpha\}}\}_{\{\alpha\}} \cup \{\mathbf{B}_{\{\alpha\}}\}_{\{\alpha\}}$$
17. `\cUnionX`  

$$\mathbf{A}_{\{\alpha\}} \cup \mathbf{B}_{\{\alpha\}}$$
18. `\cIntersPC`  $\{\alpha\}$   

$$\bigcap_{\{\alpha\} \rightarrow \{\alpha\} \rightarrow \{\alpha\}}$$
19. `\cInters`  

$$\{\mathbf{A}_{\{\alpha\}}\}_{\{\alpha\}} \cap \{\mathbf{B}_{\{\alpha\}}\}_{\{\alpha\}}$$
20. `\cIntersX`  

$$\mathbf{A}_{\{\alpha\}} \cap \mathbf{B}_{\{\alpha\}}$$
21. `\cComplPC`  $\{\alpha\}$   

$$\overline{\{\alpha\} \rightarrow \{\alpha\}}$$
22. `\cCompl`  $\{\mathbf{A}_{\{\alpha\}}\}_{\{\alpha\}}$   

$$\overline{\mathbf{A}_{\{\alpha\}}}$$
23. `\cComplX`  $\{\mathbf{A}_{\{\alpha\}}\}_{\{\alpha\}}$   

$$\overline{\mathbf{A}_{\{\alpha\}}}$$



24. `\cSetDiffPC`  $\{\alpha\}$   
 $\{\alpha\} \rightarrow \{\alpha\} \rightarrow \{\alpha\}$
25. `\cSetDiff`  
 $\{\mathbf{A}_{\cSetTy \ \alpha}\}$   
 $\{\mathbf{B}_{\cSetTy \ \alpha}\}$   
 $(\mathbf{A}_{\alpha} \setminus \mathbf{B}_{\alpha})$
26. `\cSetDiffX`  
 $\{\mathbf{A}_{\cSetTy \ \alpha}\}$   
 $\{\mathbf{B}_{\cSetTy \ \alpha}\}$   
 $\mathbf{A}_{\alpha} \setminus \mathbf{B}_{\alpha}$

## 2.10 Macros for Tuples

1. `\cTupleTyL`  
 $\alpha_1$   
 $\times$   
 $\cdots$   
 $\times$   
 $\alpha_n$   
 $(\alpha_1 \times \cdots \times \alpha_n)$
2. `\cTupleL`  
 $\{\mathbf{A}^{\{1\}}_{\cB}\}$   
 $,$   
 $\ldots$   
 $,$   
 $\{\mathbf{A}^{\{n\}}_{\cB}\}$   
 $(\mathbf{A}_o^1, \dots, \mathbf{A}_o^n)$
3. `\cFstPC`  $\{\alpha\}$   $\{\beta\}$   
 $\text{fst}_{(\alpha \times \beta) \rightarrow \alpha}$
4. `\cSndPC`  $\{\alpha\}$   $\{\beta\}$   
 $\text{snd}_{(\alpha \times \beta) \rightarrow \beta}$

## 2.11 Macros for Functions

1. `\cDomPC`  $\{\alpha\}$   $\{\beta\}$   
 $\text{dom}_{(\alpha \rightarrow \beta) \rightarrow \{\alpha\}}$
2. `\cRanPC`  $\{\alpha\}$   $\{\beta\}$   
 $\text{ran}_{(\alpha \rightarrow \beta) \rightarrow \{\beta\}}$
3. `\cSubfuneqPC`  $\{\alpha\}$   $\{\beta\}$   
 $\sqsubseteq_{(\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \beta) \rightarrow o}$
4. `\cFunCompPC`  $\{\alpha\}$   $\{\beta\}$   $\{\gamma\}$   
 $\circ_{(\alpha \rightarrow \beta) \rightarrow (\beta \rightarrow \gamma) \rightarrow (\alpha \rightarrow \gamma)}$
5. `\cFunComp`  
 $\{\mathbf{F}\}_{\{\text{cFunTyX } \{\alpha\} \{\beta\}\}}$   
 $\{\mathbf{G}\}_{\{\text{cFunTyX } \{\beta\} \{\gamma\}\}}$   
 $(\mathbf{F}_{\alpha \rightarrow \beta} \circ \mathbf{G}_{\beta \rightarrow \gamma})$
6. `\cFunCompX`  
 $\{\mathbf{F}\}_{\{\text{cFunTyX } \{\alpha\} \{\beta\}\}}$   
 $\{\mathbf{G}\}_{\{\text{cFunTyX } \{\beta\} \{\gamma\}\}}$   
 $\mathbf{F}_{\alpha \rightarrow \beta} \circ \mathbf{G}_{\beta \rightarrow \gamma}$
7. `\cRestrictPC`  $\{\alpha\}$   $\{\beta\}$   
 $|_{(\alpha \rightarrow \beta) \rightarrow \{\alpha\} \rightarrow (\alpha \rightarrow \beta)}$
8. `\cRestrict`  
 $\{\mathbf{F}\}_{\{\text{cFunTyX } \{\alpha\} \{\beta\}\}}$   
 $\{\mathbf{A}\}_{\{\text{cSetTy } \{\alpha\}\}}$   
 $(\mathbf{F}_{\alpha \rightarrow \beta} |_{\mathbf{A}_{\{\alpha\}}})$
9. `\cRestrictX`  
 $\{\mathbf{F}\}_{\{\text{cFunTyX } \{\alpha\} \{\beta\}\}}$   
 $\{\mathbf{A}\}_{\{\text{cSetTy } \{\alpha\}\}}$   
 $\mathbf{F}_{\alpha \rightarrow \beta} |_{\mathbf{A}_{\{\alpha\}}}$

## 2.12 Macros for Miscellaneous Notation

1. `\cTotal`  
 $\{\mathbf{F}\}_{\{\cFuncTyX\ \{\alpha\}\ \{\beta\}\}}$   
 $\text{TOTAL}(\mathbf{F}_{\alpha \rightarrow \beta})$
2. `\cTotalB`  
 $\{\mathbf{F}\}_{\{\cFuncTyBX\ \{\alpha\}\ \{\beta\}\ \{\gamma\}\}}$   
 $\text{TOTAL2}(\mathbf{F}_{\alpha \rightarrow \beta \rightarrow \gamma})$
3. `\cSurj`  
 $\{\mathbf{F}\}_{\{\cFuncTyX\ \{\alpha\}\ \{\beta\}\}}$   
 $\text{SURJ}(\mathbf{F}_{\alpha \rightarrow \beta})$
4. `\cSurjB`  
 $\{\mathbf{F}\}_{\{\cFuncTyBX\ \{\alpha\}\ \{\beta\}\ \{\gamma\}\}}$   
 $\text{SURJ2}(\mathbf{F}_{\alpha \rightarrow \beta \rightarrow \gamma})$
5. `\cInj`  
 $\{\mathbf{F}\}_{\{\cFuncTyX\ \{\alpha\}\ \{\beta\}\}}$   
 $\text{INJ}(\mathbf{F}_{\alpha \rightarrow \beta})$
6. `\cInjB`  
 $\{\mathbf{F}\}_{\{\cFuncTyBX\ \{\alpha\}\ \{\beta\}\ \{\gamma\}\}}$   
 $\text{INJ2}(\mathbf{F}_{\alpha \rightarrow \beta \rightarrow \gamma})$
7. `\cBij`  
 $\{\mathbf{F}\}_{\{\cFuncTyX\ \{\alpha\}\ \{\beta\}\}}$   
 $\text{BIJ}(\mathbf{F}_{\alpha \rightarrow \beta})$
8. `\cDistinctL`  
 $\{\mathbf{A}^{\{1\}}_{\{\alpha\}}\}$   
 $\text{\ldots}$   
 $\{\mathbf{A}^{\{n\}}_{\{\alpha\}}\}$   
 $\text{DISTINCT}(\mathbf{A}_{\alpha}^1, \dots, \mathbf{A}_{\alpha}^n)$

## 2.13 Macros for Quasitypes

1. `\cFunAbsQTy`  
 $\{\mathbf{x}\}$   
 $\{\mathbf{Q}_{\{\alpha\}}\}$   
 $\{\mathbf{B}_{\beta}\}$   
 $(\lambda x : \mathbf{Q}_{\{\alpha\}} . \mathbf{B}_{\beta})$
2. `\cFunAbsQTyX`  
 $\{\mathbf{x}\}$   
 $\{\mathbf{Q}_{\{\alpha\}}\}$   
 $\{\mathbf{B}_{\beta}\}$   
 $\lambda x : \mathbf{Q}_{\{\alpha\}} . \mathbf{B}_{\beta}$
3. `\cForallQTy`  
 $\{\mathbf{x}\}$   
 $\{\mathbf{Q}_{\{\alpha\}}\}$   
 $\{\mathbf{B}_{\beta}\}$   
 $(\forall x : \mathbf{Q}_{\{\alpha\}} . \mathbf{B}_{\beta})$
4. `\cForallQTyX`  
 $\{\mathbf{x}\}$   
 $\{\mathbf{Q}_{\{\alpha\}}\}$   
 $\{\mathbf{B}_{\beta}\}$   
 $\forall x : \mathbf{Q}_{\{\alpha\}} . \mathbf{B}_{\beta}$
5. `\cForallQTyB`  
 $\{\mathbf{x}\}$   
 $\{\mathbf{Q}_{\{\alpha\}}\}$   
 $\{\mathbf{y}\}$   
 $\{\mathbf{R}_{\{\beta\}}\}$   
 $\{\mathbf{B}_{\beta}\}$   
 $(\forall x : \mathbf{Q}_{\{\alpha\}}, y : \mathbf{R}_{\{\beta\}} . \mathbf{B}_{\beta})$
6. `\cForallQTyBX`  
 $\{\mathbf{x}\}$   
 $\{\mathbf{Q}_{\{\alpha\}}\}$   
 $\{\mathbf{y}\}$   
 $\{\mathbf{R}_{\{\beta\}}\}$

$\{\mathbf{B}\}_{\mathbf{C}}$

$\forall \mathbf{x} : \mathbf{Q}_{\{\alpha\}}, \mathbf{y} : \mathbf{R}_{\{\beta\}} \cdot \mathbf{B}_o$

7.  $\backslash\text{cForsomeQTy}$   
 $\{\mathbf{x}\}$   
 $\{\mathbf{Q}\}_{\{\alpha\}}$   
 $\{\mathbf{B}\}_{\mathbf{C}}$

$(\exists \mathbf{x} : \mathbf{Q}_{\{\alpha\}} \cdot \mathbf{B}_o)$

8.  $\backslash\text{cForsomeQTyX}$   
 $\{\mathbf{x}\}$   
 $\{\mathbf{Q}\}_{\{\alpha\}}$   
 $\{\mathbf{B}\}_{\mathbf{C}}$

$\exists \mathbf{x} : \mathbf{Q}_{\{\alpha\}} \cdot \mathbf{B}_o$

9.  $\backslash\text{cForsomeQTyB}$   
 $\{\mathbf{x}\}$   
 $\{\mathbf{Q}\}_{\{\alpha\}}$   
 $\{\mathbf{y}\}$   
 $\{\mathbf{R}\}_{\{\beta\}}$   
 $\{\mathbf{B}\}_{\mathbf{C}}$

$(\exists \mathbf{x} : \mathbf{Q}_{\{\alpha\}}, \mathbf{y} : \mathbf{R}_{\{\beta\}} \cdot \mathbf{B}_o)$

10.  $\backslash\text{cForsomeQTyBX}$   
 $\{\mathbf{x}\}$   
 $\{\mathbf{Q}\}_{\{\alpha\}}$   
 $\{\mathbf{y}\}$   
 $\{\mathbf{R}\}_{\{\beta\}}$   
 $\{\mathbf{B}\}_{\mathbf{C}}$

$\exists \mathbf{x} : \mathbf{Q}_{\{\alpha\}}, \mathbf{y} : \mathbf{R}_{\{\beta\}} \cdot \mathbf{B}_o$

11.  $\backslash\text{cDefDesQTy}$   
 $\{\mathbf{x}\}$   
 $\{\mathbf{Q}\}_{\{\alpha\}}$   
 $\{\mathbf{B}\}_{\mathbf{C}}$

$(\mathbf{I} \mathbf{x} : \mathbf{Q}_{\{\alpha\}} \cdot \mathbf{B}_o)$

12.  $\backslash\text{cDefDesQTyX}$   
 $\{\mathbf{x}\}$

$\{\mathbf{Q}\}_{\{\alpha\}}$   
 $\{\mathbf{B}\}_{\mathbf{B}}$

$\mathbf{I}x : \mathbf{Q}_{\{\alpha\}} \cdot \mathbf{B}_o$

13.  $\backslash\text{cIsDefInQTy}$   
 $\{\mathbf{A}\}_{\alpha}$   
 $\{\mathbf{Q}\}_{\{\alpha\}}$

$(\mathbf{A}_\alpha \downarrow \mathbf{Q}_{\{\alpha\}})$

14.  $\backslash\text{cIsDefInQTyX}$   
 $\{\mathbf{A}\}_{\alpha}$   
 $\{\mathbf{Q}\}_{\{\alpha\}}$

$\mathbf{A}_\alpha \downarrow \mathbf{Q}_{\{\alpha\}}$

15.  $\backslash\text{cIsUndInQTy}$   
 $\{\mathbf{A}\}_{\alpha}$   
 $\{\mathbf{Q}\}_{\{\alpha\}}$

$(\mathbf{A}_\alpha \uparrow \mathbf{Q}_{\{\alpha\}})$

16.  $\backslash\text{cIsUndefInQTyX}$   
 $\{\mathbf{A}\}_{\alpha}$   
 $\{\mathbf{Q}\}_{\{\alpha\}}$

$\mathbf{A}_\alpha \uparrow \mathbf{Q}_{\{\alpha\}}$

17.  $\backslash\text{cFunQTyPC } \{\alpha\} \{\beta\}$

$\rightarrow_{\{\alpha\} \rightarrow \{\beta\} \rightarrow \{\alpha \rightarrow \beta\}}$

18.  $\backslash\text{cFunQTy}$   
 $\{\mathbf{Q}\}_{\{\alpha\}}$   
 $\{\mathbf{R}\}_{\{\beta\}}$

$(\mathbf{Q}_{\{\alpha\}} \rightarrow \mathbf{R}_{\{\beta\}})$

19.  $\backslash\text{cFunQTyX}$   
 $\{\mathbf{Q}\}_{\{\alpha\}}$   
 $\{\mathbf{R}\}_{\{\beta\}}$

$\mathbf{Q}_{\{\alpha\}} \rightarrow \mathbf{R}_{\{\beta\}}$

20.  $\backslash\text{cFunQTyB}$   
 $\{\mathbf{Q}\}_{\{\alpha\}}$

$\{\mathbf{R}\}_{\{\beta\}}$   
 $\{\mathbf{S}\}_{\{\gamma\}}$

$(\mathbf{Q}_{\{\alpha\}} \rightarrow \mathbf{R}_{\{\beta\}} \rightarrow \mathbf{S}_{\{\gamma\}})$

21.  $\backslash\text{cFunQTyBX}$

$\{\mathbf{Q}\}_{\{\alpha\}}$   
 $\{\mathbf{R}\}_{\{\beta\}}$   
 $\{\mathbf{S}\}_{\{\gamma\}}$

$\mathbf{Q}_{\{\alpha\}} \rightarrow \mathbf{R}_{\{\beta\}} \rightarrow \mathbf{S}_{\{\gamma\}}$

22.  $\backslash\text{cFunQTyC}$

$\{\mathbf{Q}\}_{\{\alpha\}}$   
 $\{\mathbf{R}\}_{\{\beta\}}$   
 $\{\mathbf{S}\}_{\{\gamma\}}$   
 $\{\mathbf{T}\}_{\{\delta\}}$

$(\mathbf{Q}_{\{\alpha\}} \rightarrow \mathbf{R}_{\{\beta\}} \rightarrow \mathbf{S}_{\{\gamma\}} \rightarrow \mathbf{T}_{\{\delta\}})$

23.  $\backslash\text{cFunQTyCX}$

$\{\mathbf{Q}\}_{\{\alpha\}}$   
 $\{\mathbf{R}\}_{\{\beta\}}$   
 $\{\mathbf{S}\}_{\{\gamma\}}$   
 $\{\mathbf{T}\}_{\{\delta\}}$

$\mathbf{Q}_{\{\alpha\}} \rightarrow \mathbf{R}_{\{\beta\}} \rightarrow \mathbf{S}_{\{\gamma\}} \rightarrow \mathbf{T}_{\{\delta\}}$

24.  $\backslash\text{cProdQTyPC } \{\alpha\} \{\beta\}$

$\times_{\{\alpha\} \rightarrow \{\beta\} \rightarrow \{\alpha \times \beta\}}$

25.  $\backslash\text{cProdQTy}$

$\{\mathbf{Q}\}_{\{\alpha\}}$   
 $\{\mathbf{R}\}_{\{\beta\}}$

$(\mathbf{Q}_{\{\alpha\}} \times \mathbf{R}_{\{\beta\}})$

26.  $\backslash\text{cProdQTyX}$

$\{\mathbf{Q}\}_{\{\alpha\}}$   
 $\{\mathbf{R}\}_{\{\beta\}}$

$\mathbf{Q}_{\{\alpha\}} \times \mathbf{R}_{\{\beta\}}$

27.  $\backslash\text{cProdQTyB}$

$\{\mathbf{Q}\}_{\{\alpha\}}$

$\{\mathbf{R}\}_{\{\beta\}}$   
 $\{\mathbf{S}\}_{\{\gamma\}}$

$(\mathbf{Q}_{\{\alpha\}} \times \mathbf{R}_{\{\beta\}} \times \mathbf{S}_{\{\gamma\}})$

28.  $\backslash\text{cProdQTyBX}$   
 $\{\mathbf{Q}\}_{\{\alpha\}}$   
 $\{\mathbf{R}\}_{\{\beta\}}$   
 $\{\mathbf{S}\}_{\{\gamma\}}$

$\mathbf{Q}_{\{\alpha\}} \times \mathbf{R}_{\{\beta\}} \times \mathbf{S}_{\{\gamma\}}$

29.  $\backslash\text{cProdQTyC}$   
 $\{\mathbf{Q}\}_{\{\alpha\}}$   
 $\{\mathbf{R}\}_{\{\beta\}}$   
 $\{\mathbf{S}\}_{\{\gamma\}}$   
 $\{\mathbf{T}\}_{\{\delta\}}$

$(\mathbf{Q}_{\{\alpha\}} \times \mathbf{R}_{\{\beta\}} \times \mathbf{S}_{\{\gamma\}} \times \mathbf{T}_{\{\delta\}})$

30.  $\backslash\text{cProdQTyCX}$   
 $\{\mathbf{Q}\}_{\{\alpha\}}$   
 $\{\mathbf{R}\}_{\{\beta\}}$   
 $\{\mathbf{S}\}_{\{\gamma\}}$   
 $\{\mathbf{T}\}_{\{\delta\}}$

$\mathbf{Q}_{\{\alpha\}} \times \mathbf{R}_{\{\beta\}} \times \mathbf{S}_{\{\gamma\}} \times \mathbf{T}_{\{\delta\}}$

31.  $\backslash\text{cSetQTy}$   
 $\{\mathbf{Q}\}_{\{\alpha\}}$

$\mathcal{P}(\mathbf{Q}_{\{\alpha\}})$

32.  $\backslash\text{cTotalOn}$   
 $\{\mathbf{F}\}_{\{\alpha\} \{\beta\}}$   
 $\{\mathbf{Q}\}_{\{\alpha\}}$   
 $\{\mathbf{R}\}_{\{\beta\}}$

$\text{TOTAL-ON}(\mathbf{F}_{\alpha \rightarrow \beta}, \mathbf{Q}_{\{\alpha\}}, \mathbf{R}_{\{\beta\}})$

33.  $\backslash\text{cTotalOnB}$   
 $\{\mathbf{F}\}_{\{\alpha\} \{\beta\} \{\gamma\}}$   
 $\{\mathbf{Q}\}_{\{\alpha\}}$   
 $\{\mathbf{R}\}_{\{\beta\}}$   
 $\{\mathbf{S}\}_{\{\gamma\}}$



TOTAL-ON2( $\mathbf{F}_{\alpha \rightarrow \beta \rightarrow \gamma}$ ,  $\mathbf{Q}_{\{\alpha\}}$ ,  $\mathbf{R}_{\{\beta\}}$ ,  $\mathbf{S}_{\{\gamma\}}$ )

34. `\cSurjOn`

`{\mathbf{F}}_{\{\cFunTyX {\alpha} {\beta}\}}`  
`{\mathbf{Q}}_{\{\cSetTy {\alpha}\}}`  
`{\mathbf{R}}_{\{\cSetTy {\beta}\}}`

SURJ-ON( $\mathbf{F}_{\alpha \rightarrow \beta}$ ,  $\mathbf{Q}_{\{\alpha\}}$ ,  $\mathbf{R}_{\{\beta\}}$ )

35. `\cSurjOnB`

`{\mathbf{F}}_{\{\cFunTyBX {\alpha} {\beta} {\gamma}\}}`  
`{\mathbf{Q}}_{\{\cSetTy {\alpha}\}}`  
`{\mathbf{R}}_{\{\cSetTy {\beta}\}}`  
`{\mathbf{S}}_{\{\cSetTy {\gamma}\}}`

SURJ-ON2( $\mathbf{F}_{\alpha \rightarrow \beta \rightarrow \gamma}$ ,  $\mathbf{Q}_{\{\alpha\}}$ ,  $\mathbf{R}_{\{\beta\}}$ ,  $\mathbf{S}_{\{\gamma\}}$ )

36. `\cInjOn`

`{\mathbf{F}}_{\{\cFunTyX {\alpha} {\beta}\}}`  
`{\mathbf{Q}}_{\{\cSetTy {\alpha}\}}`

INJ-ON( $\mathbf{F}_{\alpha \rightarrow \beta}$ ,  $\mathbf{Q}_{\{\alpha\}}$ )

37. `\cInjOnB`

`{\mathbf{F}}_{\{\cFunTyBX {\alpha} {\beta} {\gamma}\}}`  
`{\mathbf{Q}}_{\{\cSetTy {\alpha}\}}`  
`{\mathbf{R}}_{\{\cSetTy {\beta}\}}`

INJ-ON2( $\mathbf{F}_{\alpha \rightarrow \beta \rightarrow \gamma}$ ,  $\mathbf{Q}_{\{\alpha\}}$ ,  $\mathbf{R}_{\{\beta\}}$ )

38. `\cBijOn`

`{\mathbf{F}}_{\{\cFunTyX {\alpha} {\beta}\}}`  
`{\mathbf{Q}}_{\{\cSetTy {\alpha}\}}`  
`{\mathbf{R}}_{\{\cSetTy {\beta}\}}`

BIJ-ON( $\mathbf{F}_{\alpha \rightarrow \beta}$ ,  $\mathbf{Q}_{\{\alpha\}}$ ,  $\mathbf{R}_{\{\beta\}}$ )

39. `\cInf {\mathbf{Q}}_{\{\cSetTy {\alpha}\}}`

INF( $\mathbf{Q}_{\{\alpha\}}$ )

40. `\cFin {\mathbf{Q}}_{\{\cSetTy {\alpha}\}}`

FIN( $\mathbf{Q}_{\{\alpha\}}$ )

41. `\cCount {\mathbf{Q}}_{\{\cSetTy {\alpha}\}}`

COUNT( $\mathbf{Q}_{\{\alpha\}}$ )

## 2.14 Macros for Dependent Types

1. `\cPiTy`  $\{\alpha\}$   $\{\beta\}$   
 $\{\alpha\} \rightarrow (\alpha \rightarrow \{\beta\}) \rightarrow \{\alpha \rightarrow \beta\}$
2. `\cPiPC`  $\{\alpha\}$   $\{\beta\}$   
 $\prod_{\{\alpha\} \rightarrow (\alpha \rightarrow \{\beta\}) \rightarrow \{\alpha \rightarrow \beta\}}$
3. `\cPi`  
 $\{\mathbf{x}\}$   
 $\{\mathbf{Q}\}_{\{\cSetTy \{\alpha\}\}}$   
 $\{\mathbf{R}\}_{\{\cSetTy \{\beta\}\}}$   
 $(\prod \mathbf{x} : \mathbf{Q}_{\{\alpha\}} \cdot \mathbf{R}_{\{\beta\}})$
4. `\cPiX`  
 $\{\mathbf{x}\}$   
 $\{\mathbf{Q}\}_{\{\cSetTy \{\alpha\}\}}$   
 $\{\mathbf{R}\}_{\{\cSetTy \{\beta\}\}}$   
 $\prod \mathbf{x} : \mathbf{Q}_{\{\alpha\}} \cdot \mathbf{R}_{\{\beta\}}$
5. `\cSigmaTy`  $\{\alpha\}$   $\{\beta\}$   
 $\{\alpha\} \rightarrow (\alpha \rightarrow \{\beta\}) \rightarrow \{\alpha \times \beta\}$
6. `\cSigmaPC`  $\{\alpha\}$   $\{\beta\}$   
 $\sum_{\{\alpha\} \rightarrow (\alpha \rightarrow \{\beta\}) \rightarrow \{\alpha \times \beta\}}$
7. `\cSigma`  
 $\{\mathbf{x}\}$   
 $\{\mathbf{Q}\}_{\{\cSetTy \{\alpha\}\}}$   
 $\{\mathbf{R}\}_{\{\cSetTy \{\beta\}\}}$   
 $(\sum \mathbf{x} : \mathbf{Q}_{\{\alpha\}} \cdot \mathbf{R}_{\{\beta\}})$
8. `\cSigmaX`  
 $\{\mathbf{x}\}$   
 $\{\mathbf{Q}\}_{\{\cSetTy \{\alpha\}\}}$   
 $\{\mathbf{R}\}_{\{\cSetTy \{\beta\}\}}$   
 $\sum \mathbf{x} : \mathbf{Q}_{\{\alpha\}} \cdot \mathbf{R}_{\{\beta\}}$

## 2.15 Macros for Sequences

1. `\cSequencesPC`  $\{\alpha\}$   $\{\beta\}$   
`sequences` <sub>$\{\alpha \rightarrow \beta\}$</sub>
2. `\cSeqQTy`  $\{\beta\}$   
 $\langle\langle\beta\rangle\rangle$
3. `\cStreamsPC`  $\{\alpha\}$   $\{\beta\}$   
`streams` <sub>$\{\alpha \rightarrow \beta\}$</sub>
4. `\cSeqInfQTy`  $\{\beta\}$   
 $\langle\beta\rangle$
5. `\cListsPC`  $\{\alpha\}$   $\{\beta\}$   
`lists` <sub>$\{\alpha \rightarrow \beta\}$</sub>
6. `\cSeqFinQTy`  $\{\beta\}$   
 $[\beta]$
7. `\cConsPC`  $\{\alpha\}$   $\{\beta\}$   
`cons` <sub>$\beta \rightarrow (\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \beta)$</sub>
8. `\cCons`  
`{\mathbf{A}}_` $\beta$   
`{\mathbf{B}}_` $\{\cFunTyX$   $\{\alpha\}$   $\{\beta\}\}$   
 $(\mathbf{A}_\beta :: \mathbf{B}_{\alpha \rightarrow \beta})$
9. `\cConsX`  
`{\mathbf{A}}_` $\beta$   
`{\mathbf{B}}_` $\{\cFunTyX$   $\{\alpha\}$   $\{\beta\}\}$   
 $\mathbf{A}_\beta :: \mathbf{B}_{\alpha \rightarrow \beta}$
10. `\cHdPC`  $\{\alpha\}$   $\{\beta\}$   
`hd` <sub>$(\alpha \rightarrow \beta) \rightarrow \beta$</sub>
11. `\cTlPC`  $\{\alpha\}$   $\{\beta\}$   
`tl` <sub>$(\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \beta)$</sub>

12. `\cNilPC`  $\{\alpha\}$   $\{\beta\}$   
 $\text{nil}_{\alpha \rightarrow \beta}$
13. `\cEmpListPC`  $\{\alpha\}$   $\{\beta\}$   
 $[\ ]_{\alpha \rightarrow \beta}$
14. `\cListL`  
 $\{\mathbf{A}^1\}_{\beta}$   
 $,$   
 $\text{\ldots}$   
 $,$   
 $\{\mathbf{A}^n\}_{\beta}$   
 $[\mathbf{A}^1_{\beta}, \dots, \mathbf{A}^n_{\beta}]$
15. `\cLenPC`  $\{\alpha\}$   $\{\beta\}$   
 $\text{len}_{(\alpha \rightarrow \beta) \rightarrow \alpha}$
16. `\cLen`  $\{\mathbf{A}\}_{\{\text{cFunTyX } \alpha \} \beta \}}$   
 $|\mathbf{A}_{\alpha \rightarrow \beta}|$
17. `\cAppendPC`  $\{\alpha\}$   $\{\beta\}$   
 $++_{(\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \beta)}$
18. `\cAppend`  
 $\{\mathbf{A}\}_{\{\text{cFunTyX } \alpha \} \beta \}}$   
 $\{\mathbf{B}\}_{\{\text{cFunTyX } \alpha \} \beta \}}$   
 $(\mathbf{A}_{\alpha \rightarrow \beta} ++ \mathbf{B}_{\alpha \rightarrow \beta})$
19. `\cAppendX`  
 $\{\mathbf{A}\}_{\{\text{cFunTyX } \alpha \} \beta \}}$   
 $\{\mathbf{B}\}_{\{\text{cFunTyX } \alpha \} \beta \}}$   
 $\mathbf{A}_{\alpha \rightarrow \beta} ++ \mathbf{B}_{\alpha \rightarrow \beta}$
20. `\cNlistsPC`  $\{\alpha\}$   $\{\beta\}$   
 $\text{nlists}_{\alpha \rightarrow \{\alpha \rightarrow \beta\}}$
21. `\cSeqNFinQTy`  $\beta$   $\{\mathbf{N}\}_{\alpha}$   
 $[\beta]^{\mathbf{N}\alpha}$

## 2.16 Macros for Reals

1. `\cMinus {\mathbf{A}}_R`  
 $(-\mathbf{A}_R)$
2. `\cMinusX {\mathbf{A}}_R`  
 $-\mathbf{A}_R$
3. `\cRecip {\mathbf{A}}_R`  
 $(\mathbf{A}_R^{-1})$
4. `\cRecipX {\mathbf{A}}_R`  
 $\mathbf{A}_R^{-1}$
5. `\cFrac {\mathbf{A}}_R {\mathbf{B}}_R`  
 $\left(\frac{\mathbf{A}_R}{\mathbf{B}_R}\right)$
6. `\cFracX {\mathbf{A}}_R {\mathbf{B}}_R`  
 $\frac{\mathbf{A}_R}{\mathbf{B}_R}$
7. `\cAbs {\mathbf{A}}_R`  
 $|\mathbf{A}_R|$
8. `\cSqrt {\mathbf{A}}_R`  
 $\sqrt{\mathbf{A}_R}$
9. `\cNorm {\mathbf{A}}_R_{\cFunTyX {R} {R}}`  
 $\|\mathbf{A}_{R \rightarrow R}\|$
10. `\cSum`  
 $\{\mathbf{i}\}$   
 $\{\mathbf{M}\}_R$   
 $\{\mathbf{N}\}_R$   $\{\mathbf{A}\}_R$   
 $\left(\sum_{i=\mathbf{M}_R}^{\mathbf{N}_R} \mathbf{A}_R\right)$
11. `\cSumX`  
 $\{\mathbf{i}\}$   
 $\{\mathbf{M}\}_R$   
 $\{\mathbf{N}\}_R$   $\{\mathbf{A}\}_R$

$$\sum_{i=M_R}^{N_R} \mathbf{A}_R$$

12. `\cProd`  
 $\{\mathbf{i}\}$   
 $\{\mathbf{M}_R\}$   
 $\{\mathbf{N}_R\}$   $\{\mathbf{A}_R\}$

$$\left( \prod_{i=M_R}^{N_R} \mathbf{A}_R \right)$$

13. `\cProdX`  
 $\{\mathbf{i}\}$   
 $\{\mathbf{M}_R\}$   
 $\{\mathbf{N}_R\}$   $\{\mathbf{A}_R\}$

$$\prod_{i=M_R}^{N_R} \mathbf{A}_R$$

14. `\cLim`  
 $\{\mathbf{x}\}$   
 $\{\mathbf{A}_R\}$   
 $\{\mathbf{B}_R\}$

$$\left( \lim_{x \rightarrow \mathbf{A}_R} \mathbf{B}_R \right)$$

15. `\cLimX`  
 $\{\mathbf{x}\}$   
 $\{\mathbf{A}_R\}$   
 $\{\mathbf{B}_R\}$

$$\lim_{x \rightarrow \mathbf{A}_R} \mathbf{B}_R$$

16. `\cRightLim`  
 $\{\mathbf{x}\}$   
 $\{\mathbf{A}_R\}$   
 $\{\mathbf{B}_R\}$

$$\left( \lim_{x \rightarrow \mathbf{A}_R^+} \mathbf{B}_R \right)$$

17. `\cRightLimX`  
 $\{\mathbf{x}\}$   
 $\{\mathbf{A}_R\}$

$\{\mathbf{B}\}_R$

$$\lim_{x \rightarrow A_R^+} \mathbf{B}_R$$

18.  $\backslash\text{cLeftLim}$   
 $\{\mathbf{x}\}$   
 $\{\mathbf{A}\}_R$   
 $\{\mathbf{B}\}_R$

$$\left( \lim_{x \rightarrow A_R^-} \mathbf{B}_R \right)$$

19.  $\backslash\text{cLeftLimX}$   
 $\{\mathbf{x}\}$   
 $\{\mathbf{A}\}_R$   
 $\{\mathbf{B}\}_R$

$$\lim_{x \rightarrow A_R^-} \mathbf{B}_R$$

20.  $\backslash\text{cLimSeq}$   
 $\{\mathbf{n}\}$   
 $\{\mathbf{B}\}_R$

$$\left( \lim_{n \rightarrow \infty} \mathbf{B}_R \right)$$

21.  $\backslash\text{cLimSeqX}$   
 $\{\mathbf{n}\}$   
 $\{\mathbf{B}\}_R$

$$\lim_{n \rightarrow \infty} \mathbf{B}_R$$

22.  $\backslash\text{cIntegral}$   
 $\{\mathbf{A}\}_R$   
 $\{\mathbf{B}\}_R$   
 $\{\mathbf{C}\}_R$   
 $\{\mathbf{x}\}$

$$\left( \int_{A_R}^{B_R} C_R dx \right)$$

23.  $\backslash\text{cIntegralX}$   
 $\{\mathbf{A}\}_R$   
 $\{\mathbf{B}\}_R$   
 $\{\mathbf{C}\}_R$   
 $\{\mathbf{x}\}$

$$\int_{\mathbf{A}_R}^{\mathbf{B}_R} \mathbf{C}_R dx$$

24. `\cExp {\mathbf{A}_R}`

$$(e^{\mathbf{A}_R})$$

25. `\cExpX {\mathbf{A}_R}`

$$e^{\mathbf{A}_R}$$

26. `\cExpB`

$$\{\mathbf{A}_R\} \\ \{\mathbf{B}_R\}$$

$$(\mathbf{A}_R^{\mathbf{B}_R})$$

27. `\cExpBX`

$$\{\mathbf{A}_R\} \\ \{\mathbf{B}_R\}$$

$$\mathbf{A}_R^{\mathbf{B}_R}$$

28. `\cLog {\mathbf{A}_R}`

$$(\log \mathbf{A}_R)$$

29. `\cLogX {\mathbf{A}_R}`

$$\log \mathbf{A}_R$$

30. `\cLoFB`

$$\{\mathbf{A}_R\} \\ \{\mathbf{B}_R\}$$

$$(\log_{\mathbf{A}_R} \mathbf{B}_R)$$

31. `\cLogBX`

$$\{\mathbf{A}_R\} \\ \{\mathbf{B}_R\}$$

$$\log_{\mathbf{A}_R} \mathbf{B}_R$$



### 3 Environments

In this section, an example of each LaTeX environment in

`alonzo-notation.tex`

is presented consisting of the LaTeX source code for the environment followed by the output it produces.

#### 3.1 Theory Definition

```
\begin{theory-def}
{Monoids}
{ $\mName{MON}$ $.}
{ $S$ $.}
{ $\cdot_{\cFuncTyBX\ S\ S\ S}$ ,  $e_S$ $.}
{
\be

\item  $\forall x,y,z \in S \quad (x \cdot y) \cdot z = x \cdot (y \cdot z)$ 
\hfill (associativity).

\item  $\forall x \in S \quad e \cdot x = x$ 
\hfill (left identity).

\item  $\forall x \in S \quad x \cdot e = x$ 
\hfill (right identity).

\ee
}
\end{theory-def}
```

#### Theory Definition 3.1 (Monoids)

**Name:** MON.

**Base types:**  $S$ .

**Constants:**  $\cdot_{S \rightarrow S \rightarrow S}$ ,  $e_S$ .

**Axioms:**

1.  $\forall x,y,z \in S \quad x \cdot (y \cdot z) = (x \cdot y) \cdot z$  (associativity).
2.  $\forall x \in S \quad e \cdot x = x$  (left identity).
3.  $\forall x \in S \quad x \cdot e = x$  (right identity).

## 3.2 Theory Extension

```
\begin{theory-ext}
{Groups}
{ $\mName{GRP}$ $.}
{ $\mName{MON}$ $.}
{(none).}
{ $\{\recip{\cdot}\}_S$ $.}
{
\be \setcounter{enumi}{3}

\item  $\forall x \in S \exists e \in S \text{ such that } x \cdot e = x$  {
e}}$ \hfill (left inverse).

\item  $\forall x \in S \exists e \in S \text{ such that } e \cdot x = x$  {
e}}$ \hfill (right inverse).

\ee
}
\end{theory-ext}
```

### Theory Extension 3.2 (Groups)

**Name:** GRP.

**Extends** MON.

**New base types:** (none).

**New constants:**  $\cdot^{-1}_{S \rightarrow S}$ .

**New axioms:**

4.  $\forall x \in S. x^{-1} \cdot x = e$  (left inverse).

5.  $\forall x \in S. x \cdot x^{-1} = e$  (right inverse).

## 3.3 Inductive Type Theory Extension

```
\begin{ind-type-theory-ext}
{Peano Arithmetic}
{ $\text{texts}\{PA-ALT\}$ $.}
{ $\mName{MIN}$ $.}
{ $\mathbb{N}$ $.}
{
```

```

\be

\item $0_N$.

\item $S_{\{\cFunTyX \{N\} \{N\}\}}$.

\ee
}
\end{ind-type-theory-ext}

```

### Inductive Type Theory Extension 3.3 (Peano Arithmetic)

**Name:** PA-ALT.

**Extends** MIN.

**New base type:**  $N$ .

**Constructors:**

1.  $0_N$ .
2.  $S_{N \rightarrow N}$ .

### 3.4 Development Definition

```

\begin{dev-def}
{\mName{PA}$ Development 1}
{\textsf{PA-dev-1}.}
{\mName{PA}$}
{
\bi

\item[] {\mName{Def1}$: $\cEqX \{1_N\} \{\cFunAppX \{S\} \{0\}\}$ \hfill (
number one)}.

\item[] {\mName{Def2}$: $\cEqX \{P_{\{\cFunTyX \{N\} \{N\}\}} \{\cFunAbsX \{
x\} \{N\} \{\cDefDesX \{y\} \{N\} \{\cEqX \{\cFunAppX \{S\} \{y\}\} \{x\}\}\}\}$ \
\hfill (predecessor function)}.

\item[] {\mName{Def3}$: $\cEqX \{+_{\{\cFunTyBX \{N\} \{N\} \{N\}\}} \{\}\}$\
\hspace*{2ex}\{\cDefDesX \{f\} \{\cFunTyBX \{N\} \{N\} \{N\}\} \{\cForallX
\{x, y\} \{N\} \{\cAndX \{\cEqX \{\cFunAppBX \{f\} \{x\} \{0\}\} \{x\}\} \{\
cEqX \{\cFunAppBX \{f\} \{x\} \{\cFunApp \{S\} \{y\}\}\} \{\cFunAppX \{S\}
\{\cFunAppB \{f\} \{x\} \{y\}\}\}\}\}\}$\ \phantom{x} \hfill (addition
function)}.

```

```

\item[] $\mName{Def4}$: $\cEqX {*_\{\cFunTyBX \{N\} \{N\} \{N\}\} \{0\}} $\hspace*{2ex}$\{\cDefDesX \{f\} \{\cFunTyBX \{N\} \{N\} \{N\}\} \{\cForallX \{x, y\} \{N\} \{\cAndX \{\cEqX \{\cFunAppBX \{f\} \{x\} \{0\}\} \{0\}\} \{\cEqX \{\cFunAppBX \{f\} \{x\} \{\cFunApp \{S\} \{y\}\}\} \{\cBinX \{\cFunAppB \{f\} \{x\} \{y\}\} \{+\} \{x\}\}\}\}\}$ $\phantom{x}$ \hfill (multiplication function).

\item[] $\mName{Thm1}$: $\cForallX \{x\} \{N\} \{\cIfX \{\cEqX \{x\} \{0\}\} \{\cIsUndefX \{\cFunApp \{P\} \{x\}\}\} \{\cIsDefX \{\cFunApp \{P\} \{x\}\}\}\} $\hfill ($P$ is almost total).

\item[] $\mName{Thm2}$: $\cForallX \{x\} \{N\} \{\cEqX \{\cFunAppX \{P\} \{\cFunApp \{S\} \{x\}\}\} \{x\}\}$ $\hfill ($P$ is a left inverse of $S$).

\item[] $\mName{Thm3}$: $\cTotalB \{+\}$ . \hfill ($+$ is total).

\item[] $\mName{Thm4}$: $\cTotalB \{*\}$ . \hfill ($*$ is total).

\ei
}
\end{dev-def}

```

### Development Definition 3.4 (PA Development 1)

**Name:** PA-dev-1.

**Bottom theory:** PA.

**Definitions and theorems:**

Def1:  $1_N = S 0$  (number one).

Def2:  $P_{N \rightarrow N} = \lambda x : N . I y : N . S y = x$  (predecessor function).

Def3:  $+_{N \rightarrow N \rightarrow N} =$   
 $I f : N \rightarrow N \rightarrow N . \forall x, y : N . f x 0 = x \wedge f x (S y) = S (f x y)$   
 (addition function).

Def4:  $*_{N \rightarrow N \rightarrow N} =$   
 $I f : N \rightarrow N \rightarrow N . \forall x, y : N . f x 0 = 0 \wedge f x (S y) = (f x y) + x$   
 (multiplication function).

Thm1:  $\forall x : N . x = 0 \mapsto (P x) \uparrow \mid (P x) \downarrow$  ( $P$  is almost total).

Thm2:  $\forall x : N . P(Sx) = x$  ( $P$  is a left inverse of  $S$ ).

Thm3: TOTAL2(+). (+ is total).

Thm4: TOTAL2(\*). (\* is total).

### 3.5 Development Extension

```

\begin{dev-ext}
{\mName{PA}$ Development 2}
{\textsf{PA-dev-2}.}
{\textsf{PA-dev-1}.}
{
\bi
\item[] {\mName{Thm5}$: $\cforallX {x,y,z} {N} {\cEqX {\cBinX {\cBin {x} {+} {y}} {+} {z}} {\cBinX {x} {+} {\cBin {y} {+} {z}}}}$ \hfill (associativity of $+$)}.

\item[] {\mName{Thm6}$: $\cforallX {x} {N} {\cBinBX {\cBinX {0} {+} {x}} {=} {x} {=} {\cBinX {x} {+} {0}}}$ \hfill ($0$ is a $+$-identity)}.

\item[] {\mName{Thm7}$: $\cforallX {x,y} {N} {\cEqX {\cBinX {x} {+} {y}} {\cBinX {y} {+} {x}}}$ \hfill (commutativity of $+$)}.

\item[] {\mName{Thm8}$: $\cforallX {x,y,z} {N} {\cEqX {\cBinX {\cBin {x} {*} {y}} {*} {z}} {\cBinX {x} {*} {\cBin {y} {*} {z}}}}$ \hfill (associativity of $*$)}.

\item[] {\mName{Thm9}$: $\cforallX {x} {N} {\cBinBX {\cBinX {1} {*} {x}} {=} {x} {=} {\cBinX {x} {*} {1}}}$ \hfill ($1$ is a $*$-identity)}.

\item[] {\mName{Thm10}$: $\cforallX {x,y} {N} {\cEqX {\cBinX {x} {*} {y}} {\cBinX {y} {*} {x}}}$ \hfill (commutativity of $*$)}.

\item[] {\mName{Thm11}$: $\cforallX {x,y,z} {N} {\cEqX {\cBinX {x} {*} {\cBin {y} {+} {z}}} {\cBinX {\cBin {x} {*} {y}} {+} {\cBin {x} {*} {z}}}}$ \phantom{x} \hfill (left distributivity)}.

\item[] {\mName{Thm12}$: $\cforallX {x,y,z} {N} {\cEqX {\cBinX {\cBin {x} {+} {y}} {*} {z}} {\cBinX {\cBin {x} {*} {z}} {+} {\cBin {x} {+} {y}}}}$ \hfill (right distributivity)}.

```

$\text{cBin } \{y\} \{*\} \{z\}\}\}\$\\ \phantom{x} \text{ \hfill (right distributivity).}$

$\text{\item[] } \$\mName{\Thm13}\$: \$\cForallX \{x\} \{N\} \{\cBinBX \{\cBinX \{0\} \{*\} \{x\}\} \{=\} \{0\} \{=\} \{\cBinX \{x\} \{*\} \{0\}\}\}\$ \text{ \hfill } (\$0\$ \text{ is an annihilator}).$

$\text{\ei}$   
 $\text{\}$   
 $\text{\end{dev-ext}}$

### Development Extension 3.5 (PA Development 2)

**Name:** PA-dev-2.

**Extends** PA-dev-1.

**New definitions and theorems:**

Thm5:  $\forall x, y, z : N . (x + y) + z = x + (y + z)$  (associativity of +).

Thm6:  $\forall x : N . 0 + x = x = x + 0$  (0 is a +-identity).

Thm7:  $\forall x, y : N . x + y = y + x$  (commutativity of +).

Thm8:  $\forall x, y, z : N . (x * y) * z = x * (y * z)$  (associativity of \*).

Thm9:  $\forall x : N . 1 * x = x = x * 1$  (1 is a \*-identity).

Thm10:  $\forall x, y : N . x * y = y * x$  (commutativity of \*).

Thm11:  $\forall x, y, z : N . x * (y + z) = (x * y) + (x * z)$   
(left distributivity).

Thm12:  $\forall x, y, z : N . (x + y) * z = (x * z) + (y * z)$   
(right distributivity).

Thm13:  $\forall x : N . 0 * x = 0 = x * 0$  (0 is an annihilator).

### 3.6 Theory Translation Definition

```

\begin{theory-trans-def}
{\mName{MON}$ to {\mName{COF}$ with {$+$}
{\textsf{MON-to-COF-{$+$}.}
{${\mName{MON}}$.}
{${\mName{COF}}$.}
{
\be

  \item $$ \mapsto R$.

\ee
}
{
\be

  \item ${\dot_{\cFuncTyBX {S} {S} {S}}}} \mapsto {+_{\cFuncTyBX {R} {R} {R}}}$$.

  \item $e_S \mapsto 0_R$.

\ee
}
\end{theory-trans-def}

```

#### Theory Translation Definition 3.6 (MON to COF with +)

**Name:** MON-to-COF-+.

**Source theory:** MON.

**Target theory:** COF.

**Base type mapping:**

1.  $S \mapsto R$ .

**Constant mapping:**

1.  $\cdot_{S \rightarrow S \rightarrow S} \mapsto +_{R \rightarrow R \rightarrow R}$ .
2.  $e_S \mapsto 0_R$ .

### 3.7 Theory Translation Extension

```

\begin{theory-trans-ext}
{\mName{GRP}$ to {\mName{COF}$ with {$+$}
{\textsf{GRP-to-COF-{$+$}.}
{\textsf{MON-to-COF-{$+$}.}
{\mName{GRP}$}.}
{\mName{COF}$}.}
{}
{
\be \setcounter{enumi}{2}

\item ${\recip{\cdot}}_{\cFunTyX \{S\} \{S\}} \mapsto \{-\}_{\cFunTyX \{R\} \{R\}}$.

\ee
}
\end{theory-trans-ext}

```

#### Theory Translation Extension 3.7 (GRP to COF with +)

**Name:** GRP-to-COF-+.

**Extends** MON-to-COF-+.

**New source theory:** GRP.

**New target theory:** COF.

**New base type mapping:**

**New constant mapping:**

3.  $\cdot^{-1}_{S \rightarrow S} \mapsto -_{R \rightarrow R}$ .

### 3.8 Development Translation Definition

```

\begin{dev-trans-def}
{\mathsf{PA}$ to {\mathsf{COF}$ 1}
{\textsf{PA-to-COF}.}
{\textsf{PA-dev-4}.}
{\textsf{COF-dev-5}.}
{
\be

```



```

\item $N \mapsto N_{\{\cSetTy {R}\}}$.

\ee
}
{
\be

\item $0_N \mapsto 0_R$.

\item $\$_{\{\cFunTyX {N} {N}\}} \mapsto \cFunAbsX {x} \{N_{\{\cSetTy {R}
\}}\} \{\cBinX {x} {+} {1}\}$.

\item $1_N \mapsto 1_R$.

\item $\$_{\{\cFunTyX {N} {N}\}} \mapsto \cFunAbsX {x} \{N_{\{\cSetTy {R}
\}}\} \{\cIfX {\cNotEqX {x} {0}} \{\cBinX {x} {-} {1}\} \{\cBotPC {R}
\}\}$.

\item $\$_{\{\cFunTyBX {N} {N} {N}\}} \mapsto \cFunAbsX {x} \{N_{\{\cSetTy
\}}\} \{\cFunAbsX {y} \{N_{\{\cSetTy {R}\}}\} \{\cBinX {x} {+} {y}\}\}$.
.

\item $\$_{\{\cFunTyBX {N} {N} {N}\}} \mapsto \cFunAbsX {x} \{N_{\{\cSetTy
\}}\} \{\cFunAbsX {y} \{N_{\{\cSetTy {R}\}}\} \{\cBinX {x} {*}\} {y}\}\}$.
.

\ee
}
\end{dev-trans-def}

```

### Development Translation Definition 3.8 (PA to COF 1)

**Name:** PA-to-COF.

**Source development:** PA-dev-4.

**Target development:** COF-dev-5.

**Base type mapping:**

1.  $N \mapsto N_{\{R\}}$ .

**Constant mapping:**

1.  $0_N \mapsto 0_R$ .

2.  $S_{N \rightarrow N} \mapsto \lambda x : N_{\{R\}} . x + 1.$
3.  $1_N \mapsto 1_R.$
4.  $P_{N \rightarrow N} \mapsto \lambda x : N_{\{R\}} . x \neq 0 \mapsto x - 1 \mid \perp_R.$
5.  $+_{N \rightarrow N \rightarrow N} \mapsto \lambda x : N_{\{R\}} . \lambda y : N_{\{R\}} . x + y.$
6.  $*_{N \rightarrow N \rightarrow N} \mapsto \lambda x : N_{\{R\}} . \lambda y : N_{\{R\}} . x * y.$

### 3.9 Development Translation Extension

```

\begin{dev-trans-ext}
{\mathsf{PA}}$ to {\mathsf{COF}}$ 2}
{\textsf{PA-to-COF-1}}$.}
{\textsf{PA-to-COF}}$.}
{\textsf{PA-dev-4}}$.}
{\textsf{COF-dev-6}}$.}
{
\be

  \item[] ${\mid}_{\cFuncTyBX \{N\} \{N\} \{cB\}} \mapsto {\mid}_{\cFuncTyBX \{R\} \{R\} \{cB\}}$.

\ee
}
\end{dev-trans-ext}

```

#### Development Translation Extension 3.9 (PA to COF 2)

- Name:** PA-to-COF-1.  
**Extends** PA-to-COF.  
**New source development:** PA-dev-4.  
**New target development:** COF-dev-6.  
**New defined constant mapping:**

$$\mid_{N \rightarrow N \rightarrow o} \mapsto \mid_{R \rightarrow R \rightarrow o}.$$

### 3.10 Definition Transportation

```

\begin{def-transport}
{Transport of Divides to  $\mathsf{COF}$ }
{\textsf{Divides-via-PA-to-COF}.}
{\textsf{PA-dev-4}.}
{\textsf{COF-dev-5}.}
{\textsf{PA-to-COF}.}
{
\bi

\item[]  $\mathsf{Def6}$ :  $\mathsf{cEqX} \{\{\mathsf{mid}\}_\{\mathsf{cFunTyBX} \{N\} \{N\} \{\mathsf{cB}\}\}\} \{\mathsf{cFunAbsX} \{x\} \{N\} \{\mathsf{cFunAbsX} \{y\} \{N\} \{\mathsf{cForsomeX} \{z\} \{N\} \{\mathsf{cEqX} \{\mathsf{cBinX} \{x\} \{*\} \{z\}\} \{y\}\}\}\}\} \mathsf{hfill} \text{(divides)}.$ 

\ei
}
{
\bi

\item[]  $\mathsf{Def6}$ -via-PA-to-COF:  $\mathsf{hspace*{2ex}} \mathsf{cEqX} \{\{\mathsf{mid}\}_\{\mathsf{cFunTyBX} \{R\} \{R\} \{\mathsf{cB}\}\}\} \{\mathsf{cFunAbsX} \{x\} \{N_\{\mathsf{cSetTy} \{R\}\}\} \{\mathsf{cFunAbsX} \{y\} \{N_\{\mathsf{cSetTy} \{R\}\}\}\} \{\mathsf{cForsomeX} \{z\} \{N_\{\mathsf{cSetTy} \{R\}\}\} \{\mathsf{cEqX} \{\mathsf{cBinX} \{x\} \{*\} \{z\}\} \{y\}\}\}\} \mathsf{hfill} \text{(divides)}.$ 

\ei
}
{\textsf{COF-dev-6}.}
{\textsf{PA-to-COF-1}.}
\end{def-transport}

```

#### Definition Transportation 3.10 (Transport of Divides to COF)

**Name:** Divides-via-PA-to-COF.

**Source development:** PA-dev-4.

**Target development:** COF-dev-5.

**Development morphism:** PA-to-COF.

**Definition:**

$$\mathsf{Def6}: \lfloor_{N \rightarrow N \rightarrow o} = \lambda x : N . \lambda y : N . \exists z : N . x * z = y \quad \text{(divides)}.$$

**Transported definition:**

Def6-via-PA-to-COF:

$$\mid_{R \rightarrow R \rightarrow o} = \lambda x : N_{\{R\}} . \lambda y : N_{\{R\}} . \exists z : N_{\{R\}} . x * z = y \quad (\text{divides}).$$

**New target development:** COF-dev-6.

**New development morphism:** PA-to-COF-1.

### 3.11 Theorem Transportation

```
\begin{thm-transport}
{Transport of 0 is Top to  $\mathsf{COF}$ }
{\textsf{0-is-Top-via-PA-to-COF-1}.}
{\textsf{PA-dev-4}.}
{\textsf{COF-dev-6}.}
{\textsf{PA-to-COF-1}.}
{
\bi

\item[]  $\mathsf{Thm22}$ :  $\mathsf{cForallX \{x\} \{N\} \{cBinX \{x\} \{\mid\}_\{cFunTyBX \{N\} \{N\} \{cB\}\} \{0\}}$   $\mathsf{hfill}$  ( $\mathsf{0}$  is top).

\ei
}
{
\bi

\item[]  $\mathsf{Thm22}$ -via-PA-to-COF-1:
\hspace*{2ex} $\mathsf{cForallX \{x\} \{N_{\{cSetTy \{R\}\}} \{cBinX \{x\} \{\mid\}_\{cFunTyBX \{R\} \{R\} \{cB\}\} \{0\}}$   $\mathsf{hfill}$  ( $\mathsf{0}$  is top).

\ei
}
{\textsf{COF-dev-7}.}
\end{thm-transport}
```

#### Theorem Transportation 3.11 (Transport of 0 is Top to COF)

**Name:** 0-is-Top-via-PA-to-COF-1.

**Source development:** PA-dev-4.

**Target development:** COF-dev-6.

**Development morphism:** PA-to-COF-1.

**Theorem:**

Thm22:  $\forall x : N . x \mid_{N \rightarrow N \rightarrow o} 0$  (0 is top).

**Transported theorem:**

Thm22-via-PA-to-COF-1:  $\forall x : N_{\{R\}} . x \mid_{R \rightarrow R \rightarrow o} 0$  (0 is top).

**New target development:** COF-dev-7.

### 3.12 Group Transportation

```

\begin{group-transport}
{\small Transport of  $\mid$  and ‘0 is Top’ to  $\mathsf{COF}$ }
{\textsf{Divides-and-0-is-top-via-PA-to-COF}.}
{\textsf{PA-dev-4}.}
{\textsf{COF-dev-5}.}
{\textsf{PA-to-COF}.}
{
\bi

\item[]  $\mathsf{Def6}$ :  $\mathsf{EqX} \{ \mid \}_ { \mathsf{FunTyBX} \{ N \} \{ N \} \{ \mathsf{CB} \} } \{ \mathsf{FunAbsX} \{ x \} \{ N \} \{ \mathsf{FunAbsX} \{ y \} \{ N \} \{ \mathsf{ForsomeX} \{ z \} \{ N \} \{ \mathsf{EqX} \{ \mathsf{BinX} \{ x \} \{ * \} \{ z \} \} \{ y \} \} \} \}$  \hfill (divides).

\item[]  $\mathsf{Thm22}$ :  $\mathsf{ForallX} \{ x \} \{ N \} \{ \mathsf{BinX} \{ x \} \{ \mid \}_ { \mathsf{FunTyBX} \{ N \} \{ N \} \{ \mathsf{CB} \} } \{ 0 \} \}$  \hfill ( $0$  is top).

\ei
}
{
\bi

\item[] \textsf{Def6-via-PA-to-COF}: \\\
\hspace*{2ex}  $\mathsf{EqX} \{ \mid \}_ { \mathsf{FunTyBX} \{ R \} \{ R \} \{ \mathsf{CB} \} } \{ \mathsf{FunAbsX} \{ x \} \{ N_{\mathsf{SetTy} \{ R \} } \} \{ \mathsf{FunAbsX} \{ y \} \{ N_{\mathsf{SetTy} \{ R \} } \} \} \}$ 
\hspace*{2ex}  $\{ \mathsf{ForsomeX} \{ z \} \{ N_{\mathsf{SetTy} \{ R \} } \} \} \{ \mathsf{EqX} \{ \mathsf{BinX} \{ x \} \{ * \} \{ z \} \} \{ y \} \} \}$ 
\hfill (divides).

\item[] \textsf{Thm22-via-PA-to-COF}:  $\mathsf{ForallX} \{ x \} \{ N_{\mathsf{SetTy} \{ R \} } \} \{ \mathsf{BinX} \{ x \} \{ \mid \}_ { \mathsf{FunTyBX} \{ R \} \{ R \} \{ \mathsf{CB} \} } \{ 0 \} \}$ 
\hfill ( $0$  is top).

```

```

\ei
}
{\textsf{COF-dev-6}.}
{\textsf{PA-to-COF-1}.}
\end{group-transport}

```

### Group Transportation 3.12 (Transport of $|$ and “0 is Top” to COF)

**Name:** Divides-and-0-is-top-via-PA-to-COF.

**Source development:** PA-dev-4.

**Target development:** COF-dev-5.

**Development morphism:** PA-to-COF.

**Definitions and theorems:**

Def6:  $|_{N \rightarrow N \rightarrow o} = \lambda x : N . \lambda y : N . \exists z : N . x * z = y$  (divides).

Thm22:  $\forall x : N . x |_{N \rightarrow N \rightarrow o} 0$  (0 is top).

**Transported definitions and theorems:**

Def6-via-PA-to-COF:

$|_{R \rightarrow R \rightarrow o} = \lambda x : N_{\{R\}} . \lambda y : N_{\{R\}} . \exists z : N_{\{R\}} . x * z = y$  (divides).

Thm22-via-PA-to-COF:  $\forall x : N_{\{R\}} . x |_{R \rightarrow R \rightarrow o} 0$  (0 is top).

**New target development:** COF-dev-6.

**New development morphism:** PA-to-COF-1.

## 4 LaTeX Source File

```

%% LaTeX Macros for Alonzo Notation
%%
%% William M. Farmer
%%
%% McMaster University
%%
%% July 14, 2024

%% Requires:

```

```

\usepackage{amssymb}
\usepackage{amsmath}
\usepackage{phonetic}
\usepackage{xcolor}

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% MISCELLANEOUS MACROS

\newcommand{\mName}[1]{\mathsf{#1}}
\newcommand{\mSet}[1]{\{ #1 \}}
\newcommand{\mTuple}[1]{( #1 )}
\newcommand{\mList}[1]{[ #1 ]}
\newcommand{\mSeq}[1]{\langle #1 \rangle}
\newcommand{\mSeqlike}[1]{\mSeq{\!\mSeq{#1}\!}}
\newcommand{\mAbs}[1]{\lvert #1 \rvert}
\newcommand{\mNorm}[1]{\lVert #1 \rVert}
\newcommand{\mDot}{\mathrel{.}}

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% FORMAL NOTATION

% Types

\newcommand{\fBoolTy}{\mName{BoolTy}}
\newcommand{\fBaseTy}[1]{\mName{BaseTy}(#1)}
\newcommand{\fFunTy}[2]{\mName{FunTy}(#1,#2)}
\newcommand{\fProdTy}[2]{\mName{ProdTy}(#1,#2)}

% Expressions

\newcommand{\fVar}[2]{\mName{Var}(#1,#2)}
\newcommand{\fCon}[2]{\mName{Con}(#1,#2)}
\newcommand{\fEq}[2]{\mName{Eq}(#1,#2)}
\newcommand{\fFunApp}[2]{\mName{FunApp}(#1,#2)}
\newcommand{\fFunAbs}[3]{\mName{FunAbs}(\fVar{#1}{#2},#3)}
\newcommand{\fDefDes}[3]{\mName{DefDes}(\fVar{#1}{#2},#3)}
\newcommand{\fOrdPair}[2]{\mName{OrdPair}(#1,#2)}

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% COMPACT NOTATION

% Types

```

```

\newcommand{\cBoolTy}{\omicron}
\newcommand{\cB}{\cBoolTy}
\newcommand{\cBaseTy}[1]{#1}
\newcommand{\cFunTy}[2]{({#1} \rightarrow {#2})}
\newcommand{\cFunTyX}[2]{#{#1} \rightarrow {#2}}
\newcommand{\cFunTyB}[3]{\cFunTy {#1} {\cFunTyX {#2} {#3}}}
\newcommand{\cFunTyBX}[3]{\cFunTyX {#1} {\cFunTyX {#2} {#3}}}
\newcommand{\cFunTyC}[4]{\cFunTy {#1} {\cFunTyX {#2} {\cFunTyX {#3}
{#4}}}}
\newcommand{\cFunTyCX}[4]{\cFunTyX {#1} {\cFunTyX {#2} {\cFunTyX
{#3} {#4}}}}
\newcommand{\cProdTy}[2]{({#1} \times {#2})}
\newcommand{\cProdTyX}[2]{#{#1} \times {#2}}
\newcommand{\cProdTyB}[3]{\cProdTy {#1} {\cProdTyX {#2} {#3}}}
\newcommand{\cProdTyBX}[3]{\cProdTyX {#1} {\cProdTyX {#2} {#3}}}
\newcommand{\cProdTyC}[4]{\cProdTy {#1} {\cProdTyX {#2} {\cProdTyX
{#3} {#4}}}}
\newcommand{\cProdTyCX}[4]{\cProdTyX {#1} {\cProdTyX {#2} {\cProdTyX
{#3} {#4}}}}

```

% Expressions

```

\newcommand{\cVar}[2]{({#1} : {#2})}
\newcommand{\cVarY}[2]{#1}
\newcommand{\cCon}[2]{#{#1}_#{#2}}
\newcommand{\cConY}[2]{#1}
\newcommand{\cEq}[2]{({#1} = {#2})}
\newcommand{\cEqX}[2]{#{#1} = {#2}}
\newcommand{\cFunApp}[2]{(#1\, #2)}
\newcommand{\cFunAppX}[2]{#1\, #2}
\newcommand{\cFunAppB}[3]{(\cFunAppX {\cFunAppX {#1} {#2}} {#3})}
\newcommand{\cFunAppBX}[3]{\cFunAppX {\cFunAppX {#1} {#2}} {#3}}
\newcommand{\cFunAppC}[4]{(\cFunAppX {\cFunAppX {\cFunAppX
{#1}{#2}}{#3}}{#4})}
\newcommand{\cFunAppCX}[4]{\cFunAppX {\cFunAppX {\cFunAppX
{#1}{#2}}{#3}}{#4}}
\newcommand{\cFunAbs}[3]{(\lambda\, #1 : #2 \mDot #3)}
\newcommand{\cFunAbsX}[3]{\lambda\, #1 : #2 \mDot #3}
\newcommand{\cDefDes}[3]{(\mathrm{I}\, #1 : #2 \mDot #3)}
\newcommand{\cDefDesX}[3]{\mathrm{I}\, #1 : #2 \mDot #3}
\newcommand{\cOrdPair}[2]{(#1, #2)}

```

% Boolean Operators



```

\newcommand{\cTPC}{T_ \cB}
\newcommand{\cT}{\cTPC}
\newcommand{\cFPC}{F_ \cB}
\newcommand{\cF}{\cFPC}
\newcommand{\cAndPC}{\wedge_ {\cFunTyBX{\cB}{\cB}{\cB}}}
\newcommand{\cAnd}[2]{({#1} \wedge {#2})}
\newcommand{\cAndX}[2]{#{#1} \wedge {#2}}
\newcommand{\cAndB}[3]{\cAnd {#1} {\cAndX {#2} {#3}}}
\newcommand{\cAndBX}[3]{\cAndX {#1} {\cAndX {#2} {#3}}}
\newcommand{\cAndL}[1]{(#1)} % separator is $\And$
\newcommand{\cAndLX}[1]{#1} % separator is $\And$
\newcommand{\cImpliesPC}{\rightarrow_ {\cFunTyBX {\cB} {\cB} {\cB}}}
\newcommand{\cImplies}[2]{({#1} \rightarrow {#2})}
\newcommand{\cImpliesX}[2]{#{#1} \rightarrow {#2}}
\newcommand{\cNegPC}{\neg_ {\cFunTyX{\cB}{\cB}}}
\newcommand{\cNeg}[1]{(\neg{#1})}
\newcommand{\cNegX}[1]{\neg{#1}}
\newcommand{\cOrPC}{\vee_ {\cFunTyBX{\cB}{\cB}{\cB}}}
\newcommand{\cOr}[2]{({#1} \vee {#2})}
\newcommand{\cOrX}[2]{#{#1} \vee {#2}}
\newcommand{\cOrB}[3]{\cOr {#1} {\cOrX {#2} {#3}}}
\newcommand{\cOrBX}[3]{\cOrX {#1} {\cOrX {#2} {#3}}}
\newcommand{\cOrL}[1]{(#1)} % separator is $\Or$
\newcommand{\cOrLX}[1]{#1} % separator is $\Or$

% Binary Operators

\newcommand{\cBin}[3]{({#1} \mathrel{#2} {#3})}
\newcommand{\cBinX}[3]{#{#1} \mathrel{#2} {#3}}
\newcommand{\cBinB}[5]{({#1} \mathrel{#2} {#3} \mathrel{#4} {#5})}
\newcommand{\cBinBX}[5]{#{#1} \mathrel{#2} {#3} \mathrel{#4} {#5}}
\newcommand{\cIff}[2]{({#1} \Leftrightarrow {#2})}
\newcommand{\cIffX}[2]{#{#1} \Leftrightarrow {#2}}
\newcommand{\cNotEq}[2]{({#1} \not= {#2})}
\newcommand{\cNotEqX}[2]{#{#1} \not= {#2}}

% Quantifiers

\newcommand{\cForall}[3]{(\forall\, , #1 : #2 \mDot #3)}
\newcommand{\cForallX}[3]{\forall\, , #1 : #2 \mDot #3}
\newcommand{\cForallB}[5]{(\forall\, , #1 : #2,\, , #3 : #4 \mDot #5)}
\newcommand{\cForallBX}[5]{\forall\, , #1 : #2,\, , #3 : #4 \mDot #5}
\newcommand{\cForallC}[7]{(\forall\, , #1 : #2,\, , #3 : #4,\, , #5 : #6 \mDot #7)}
\newcommand{\cForallCX}[7]{\forall\, , #1 : #2,\, , #3 : #4,\, , #5 : #6 \mDot #7}

```

```

mDot #7}
\newcommand{\cForsome}[3]{(\exists\, #1 : #2 \mDot #3)}
\newcommand{\cForsomeX}[3]{\exists\, #1 : #2 \mDot #3}
\newcommand{\cForsomeB}[5]{(\exists\, #1 : #2,\, #3 : #4 \mDot #5)}
\newcommand{\cForsomeBX}[5]{\exists\, #1 : #2,\, #3 : #4 \mDot #5}
\newcommand{\cForsomeC}[7]{(\exists\, #1 : #2,\, #3 : #4,\, #5 : #6
\mDot #7)}
\newcommand{\cForsomeCX}[7]{\exists\, #1 : #2,\, #3 : #4,\, #5 : #6
\mDot #7}
\newcommand{\cForsomeUnique}[3]{(\exists!\, #1 : #2 \mDot #3)}
\newcommand{\cForsomeUniqueX}[3]{\exists!\, #1 : #2 \mDot #3}

% Definedness

\newcommand{\cBotPC}[1]{\bot_{\{#1\}}}
\newcommand{\cEmpFunPC}[2]{\Delta_{\{ \cFunTyX {#1} {#2} \}}}
\newcommand{\cIsDef}[1]{(#1\downarrow)}
\newcommand{\cIsDefX}[1]{#1\downarrow}
\newcommand{\cIsUndef}[1]{(#1\uparrow)}
\newcommand{\cIsUndefX}[1]{#1\uparrow}
\newcommand{\cQuasiEq}[2]{(#1 \simeq {#2})}
\newcommand{\cQuasiEqX}[2]{#1 \simeq {#2}}
\newcommand{\cNotQuasiEq}[2]{(#1 \not\simeq {#2})}
\newcommand{\cNotQuasiEqX}[2]{#1 \not\simeq {#2}}
\newcommand{\cIfThenElse}[3]{\mName{IF}(#1,#2,#3)}
\newcommand{\cIf}[3]{(#1 \mapsto #2 \mid #3)}
\newcommand{\cIfX}[3]{#1 \mapsto #2 \mid #3}

% Sets

\newcommand{\cSetTy}[1]{\mSet{#1}}
\newcommand{\cIn}[2]{({#1} \in {#2})}
\newcommand{\cInX}[2]{#1 \in {#2}}
\newcommand{\cNotIn}[2]{({#1} \not\in {#2})}
\newcommand{\cNotInX}[2]{#1 \not\in {#2}}
\newcommand{\cSet}[3]{\mSet{{#1} : {#2}} \mid {#3}}
\newcommand{\cEmpSetPC}[1]{\emptyset_{\{ \cSetTy {#1} \}}}
\newcommand{\cEmpSetAltPC}[1]{\mSet{\,}_\{ \cSetTy {#1} \}}
\newcommand{\cUnivSetPC}[1]{U_{\{ \cSetTy {#1} \}}}
\newcommand{\cFinSet}[2]{\textsf{${#1}$-}${#2}$-SET}}
\newcommand{\cFinSetL}[1]{\mSet{#1}} % separator is ","
\newcommand{\cSubseteqPC}[1]{\subseq_{\{ \cFunTyBX { \cSetTy #1 } { \cSetTy #1 } { \cB } \}}}
\newcommand{\cSubseteq}[2]{\cBin {#1} { \subseq } {#2}}
\newcommand{\cSubseteqX}[2]{\cBinX {#1} { \subseq } {#2}}

```

```

\newcommand{\cUnionPC}[1]{\cup_{\cFunTyBX {\cSetTy #1} {\cSetTy #1}
  {\cSetTy #1}}}
\newcommand{\cUnion}[2]{\cBin {#1} {\cup} {#2}}
\newcommand{\cUnionX}[2]{\cBinX {#1} {\cup} {#2}}
\newcommand{\cIntersPC}[1]{\cap_{\cFunTyBX {\cSetTy #1} {\cSetTy #1}
  {\cSetTy #1}}}
\newcommand{\cInters}[2]{\cBin {#1} {\cap} {#2}}
\newcommand{\cIntersX}[2]{\cBinX {#1} {\cap} {#2}}
\newcommand{\cComplPC}[1]{\overline{\,\cdot\,}_{\cFunTyX {\cSetTy
  #1} {\cSetTy #1}}}
\newcommand{\cCompl}[1]{\big(\,\overline{#1}\,\big)}
\newcommand{\cComplX}[1]{\overline{#1}}
\newcommand{\cSetDiffPC}[1]{\setminus_{\cFunTyBX {\cSetTy #1} {\c
  SetTy #1} {\cSetTy #1}}}
\newcommand{\cSetDiff}[2]{\cBin {#1} {\setminus} {#2}}
\newcommand{\cSetDiffX}[2]{\cBinX {#1} {\setminus} {#2}}

% Tuples

\newcommand{\cTupleTyL}[1]{(#1)} % separator is $\times$
\newcommand{\cTupleL}[1]{(#1)} % separator is ","
\newcommand{\cFstPC}[2]{\mName{fst}_{\cFunTyX {\cProdTy {#1} {#2}}
  {#1}}}
\newcommand{\cSndPC}[2]{\mName{snd}_{\cFunTyX {\cProdTy {#1} {#2}}
  {#2}}}

% Functions

\newcommand{\cDomPC}[2]{\mName{dom}_{\cFunTyX {\cFunTy {#1} {#2}} {\c
  SetTy {#1}}}
\newcommand{\cRanPC}[2]{\mName{ran}_{\cFunTyX {\cFunTy {#1} {#2}} {\c
  SetTy {#2}}}
\newcommand{\cSubfuneqPC}[2]{\sqsubseteq_{\cFunTyBX {\cFunTy {#1}
  {#2}} {\cFunTy {#1} {#2}} {\cB}}
\newcommand{\cFunCompPC}[3]{\circ_{\cFunTyBX {\cFunTy {#1} {#2}} {\c
  FunTy {#2} {#3}} {\cFunTy {#1} {#3}}}
\newcommand{\cFunComp}[2]{({#1} \circ {#2})}
\newcommand{\cFunCompX}[2]{#1 \circ {#2}}
\newcommand{\cRestrictPC}[2]{|_{\cFunTyBX {\cFunTy {#1} {#2}} {\c
  SetTy {#1}} {\cFunTy {#1} {#2}}}
\newcommand{\cRestrict}[2]{(#1 |_{#2})}
\newcommand{\cRestrictX}[2]{#1 |_{#2}}

% Miscellaneous Notation

```

```

\newcommand{\cTotal}[1]{\mName{TOTAL}(#1)}
\newcommand{\cTotalB}[1]{\mName{TOTAL2}(#1)}
\newcommand{\cSurj}[1]{\mName{SURJ}(#1)}
\newcommand{\cSurjB}[1]{\mName{SURJ2}(#1)}
\newcommand{\cInj}[1]{\mName{INJ}(#1)}
\newcommand{\cInjB}[1]{\mName{INJ2}(#1)}
\newcommand{\cBij}[1]{\mName{BIJ}(#1)}
\newcommand{\cDistinctL}[1]{\mName{DISTINCT}(#1)} % separator is ","

% Quasitypes

\newcommand{\cFunAbsQTy}[3]{\cFunAbs {#1} {#2} {#3}}
\newcommand{\cFunAbsQTyX}[3]{\cFunAbsX {#1} {#2} {#3}}
\newcommand{\cForallQTy}[3]{\cForall {#1} {#2} {#3}}
\newcommand{\cForallQTyX}[3]{\cForallX {#1} {#2} {#3}}
\newcommand{\cForallQTyB}[5]{\cForallB {#1} {#2} {#3} {#4} {#5}}
\newcommand{\cForallQTyBX}[5]{\cForallBX {#1} {#2} {#3} {#4} {#5}}
\newcommand{\cForsomeQTy}[3]{\cForsome {#1} {#2} {#3}}
\newcommand{\cForsomeQTyX}[3]{\cForsomeX {#1} {#2} {#3}}
\newcommand{\cForsomeQTyB}[5]{\cForsomeB {#1} {#2} {#3} {#4} {#5}}
\newcommand{\cForsomeQTyBX}[5]{\cForsomeBX {#1} {#2} {#3} {#4} {#5}}
\newcommand{\cDefDesQTy}[3]{\cDefDes {#1} {#2} {#3}}
\newcommand{\cDefDesQTyX}[3]{\cDefDesX {#1} {#2} {#3}}
\newcommand{\cIsDefInQTy}[2]{({#1} \downarrow {#2})}
\newcommand{\cIsDefInQTyX}[2]{({#1} \downarrow {#2})}
\newcommand{\cIsUndefInQTy}[2]{({#1} \uparrow {#2})}
\newcommand{\cIsUndefInQTyX}[2]{({#1} \uparrow {#2})}
\newcommand{\cFunQTyPC}[2]{\rightarrow_{\cFunTyBX {\cSetTy {#1}} {\cSetTy {#2}} {\cSetTy {\cFunTyX {#1} {#2}}}}
\newcommand{\cFunQTy}[2]{\cFunTy {#1} {#2}}
\newcommand{\cFunQTyX}[2]{\cFunTyX {#1} {#2}}
\newcommand{\cFunQTyB}[3]{\cFunTyB {#1} {#2} {#3}}
\newcommand{\cFunQTyBX}[3]{\cFunTyBX {#1} {#2} {#3}}
\newcommand{\cFunQTyC}[4]{\cFunTyC {#1} {#2} {#3} {#4}}
\newcommand{\cFunQTyCX}[4]{\cFunTyCX {#1} {#2} {#3} {#4}}
\newcommand{\cProdQTyPC}[2]{\times_{\cFunTyBX {\cSetTy {#1}} {\cSetTy {#2}} {\cSetTy {\cProdTyX {#1} {#2}}}}
\newcommand{\cProdQTy}[2]{\cProdTy {#1} {#2}}
\newcommand{\cProdQTyX}[2]{\cProdTyX {#1} {#2}}
\newcommand{\cProdQTyB}[3]{\cProdTyB {#1} {#2} {#3}}
\newcommand{\cProdQTyBX}[3]{\cProdTyBX {#1} {#2} {#3}}
\newcommand{\cProdQTyC}[4]{\cProdTyC {#1} {#2} {#3} {#4}}
\newcommand{\cProdQTyCX}[4]{\cProdTyCX {#1} {#2} {#3} {#4}}
\newcommand{\cSetQTy}[1]{\cal P}{#1}}
\newcommand{\cTotalOn}[3]{\textsf{TOTAL-ON}(#1,#2,#3)}

```

```

\newcommand{\cTotalOnB}[4]{\textsf{TOTAL-ON2}(\#1,\#2,\#3,\#4)}
\newcommand{\cSurjOn}[3]{\textsf{SURJ-ON}(\#1,\#2,\#3)}
\newcommand{\cSurjOnB}[4]{\textsf{SURJ-ON2}(\#1,\#2,\#3,\#4)}
\newcommand{\cInjOn}[2]{\textsf{INJ-ON}(\#1,\#2)}
\newcommand{\cInjOnB}[3]{\textsf{INJ-ON2}(\#1,\#2,\#3)}
\newcommand{\cBijOn}[3]{\textsf{BIJ-ON}(\#1,\#2,\#3)}
\newcommand{\cInf}[1]{\mName{INF}(\#1)}
\newcommand{\cFin}[1]{\mName{FIN}(\#1)}
\newcommand{\cCount}[1]{\mName{COUNT}(\#1)}

% Dependent Quasitypes

\newcommand{\cPiTy}[2]{\cFunTyBX {\cSetTy {\#1}} {\cFunTy {\#1} {\cSetTy {\#2}}}} {\cSetTy {\cFunTyX {\#1} {\#2}}}}
\newcommand{\cPiPC}[2]{\Pi_{\cPiTy {\#1} {\#2}}}
\newcommand{\cPi}[3]{(\Pi\, ,\#1 : \#2 \mDot \#3)}
\newcommand{\cPiX}[3]{\Pi\, ,\#1 : \#2 \mDot \#3}
\newcommand{\cSigmaTy}[2]{\cFunTyBX {\cSetTy {\#1}} {\cFunTy {\#1} {\cSetTy {\#2}}}} {\cSetTy {\cProdTyX {\#1} {\#2}}}}
\newcommand{\cSigmaPC}[2]{\Sigma_{\cSigmaTy {\#1} {\#2}}}
\newcommand{\cSigma}[3]{(\Sigma\, ,\#1 : \#2 \mDot \#3)}
\newcommand{\cSigmaX}[3]{\Sigma\, ,\#1 : \#2 \mDot \#3}

% Sequences

\newcommand{\cSequencesPC}[2]{\mName{sequences}_{\cSetTy {\cFunTyX {\#1} {\#2}}}}
\newcommand{\cSeqQTy}[1]{\mSeqlike{\#1}}
\newcommand{\cStreamsPC}[2]{\mName{streams}_{\cSetTy {\cFunTyX {\#1} {\#2}}}}
\newcommand{\cSeqInfQTy}[1]{\mSeq{\#1}}
\newcommand{\cListsPC}[2]{\mName{lists}_{\cSetTy {\cFunTyX {\#1} {\#2}}}}
\newcommand{\cSeqFinQTy}[1]{\mList{\#1}}
\newcommand{\cConsPC}[2]{\mName{cons}_{\cFunTyBX {\#2} {\cFunTy {\#1} {\#2}}}} {\cFunTy {\#1} {\#2}}}}
\newcommand{\cCons}[2]{({\#1} :: {\#2})}
\newcommand{\cConsX}[2]{{\#1} :: {\#2}}
\newcommand{\cHdPC}[2]{\mName{hd}_{\cFunTyX {\cFunTy {\#1} {\#2}}}} {\#2}}
\newcommand{\cTlPC}[2]{\mName{tl}_{\cFunTyX {\cFunTy {\#1} {\#2}}}} {\cFunTy {\#1} {\#2}}}}
\newcommand{\cNilPC}[2]{\mName{nil}_{\cFunTyX {\#1} {\#2}}}}
\newcommand{\cEmpListPC}[2]{\mList{\;}}_{\cFunTyX {\#1} {\#2}}}}
\newcommand{\cListL}[1]{\mList{\#1}} % separator is ", "

```

```

\newcommand{\cLenPC}[2]{\mName{len}_{\cFunTyX {\cFunTy {#1} {#2}}
  {#1}}}
\newcommand{\cLen}[1]{\mAbs {#1}}
\newcommand{\cAppendPC}[2]{\mName{++}_{\cFunTyBX {\cFunTy {#1} {#2}}
  {\cFunTy {#1} {#2}} {\cFunTy {#1} {#2}}}}
\newcommand{\cAppend}[2]{({#1} \mathrel{++} {#2})}
\newcommand{\cAppendX}[2]{#{#1} \mathrel{++} {#2}}
\newcommand{\cNlistsPC}[2]{\mName{nlists}_{\cFunTyX {#1} {\cSetTy {\cFunTyX {#1} {#2}}}}
\newcommand{\cSeqNFinQTy}[2]{\mList{#1}^{#2}}

% Real Numbers

\newcommand{\cMinus}[1]{(-{#1})}
\newcommand{\cMinusX}[1]{-#{#1}}
\newcommand{\cRecip}[1]{\big( {#1}^{-1} \big)}
\newcommand{\cRecipX}[1]{#{#1}^{-1}}
\newcommand{\cFrac}[2]{\big( \frac {#1} {#2} \big)}
\newcommand{\cFracX}[2]{\frac {#1} {#2}}
\newcommand{\cAbs}[1]{\mAbs {#1}}
\newcommand{\cSqrt}[1]{\sqrt {#1}}
\newcommand{\cNorm}[1]{\mNorm {#1}}
\newcommand{\cSum}[4]{\Big( \sum\limits_{{#1} = {#2}}^{#3} {#4} \Big)}
\newcommand{\cSumX}[4]{\sum\limits_{{#1} = {#2}}^{#3} {#4}}
\newcommand{\cProd}[4]{\Big( \prod\limits_{{#1} = {#2}}^{#3} {#4} \Big)}
\newcommand{\cProdX}[4]{\prod\limits_{{#1} = {#2}}^{#3} {#4}}
\newcommand{\cLim}[3]{\Big( \lim\limits_{{#1} \to {#2}} {#3} \Big)}
\newcommand{\cLimX}[3]{\lim\limits_{{#1} \to {#2}} {#3}}
\newcommand{\cRightLim}[3]{\Big( \lim\limits_{{#1} \to {#2}^+} {#3} \Big)}
\newcommand{\cRightLimX}[3]{\lim\limits_{{#1} \to {#2}^+} {#3}}
\newcommand{\cLeftLim}[3]{\Big( \lim\limits_{{#1} \to {#2}^-} {#3} \Big)}
\newcommand{\cLeftLimX}[3]{\lim\limits_{{#1} \to {#2}^-} {#3}}
\newcommand{\cLimSeq}[2]{\Big( \lim\limits_{{#1} \to \infty} {#2} \Big)}
\newcommand{\cLimSeqX}[2]{\lim\limits_{{#1} \to \infty} {#2}}
\newcommand{\cIntegral}[4]{\Big( \int_{{#1}^{#2}} {#3} \, d{#4} \Big)}
\newcommand{\cIntegralX}[4]{\int_{{#1}^{#2}} {#3} \, d{#4}}
\newcommand{\cExp}[1]{(\mName{e}^{#1})}
\newcommand{\cExpX}[1]{\mName{e}^{#1}}
\newcommand{\cExpB}[2]{({#1}^{#2})}
\newcommand{\cExpBX}[2]{#{#1}^{#2}}

```

```

\newcommand{\cLog}[1]{(\mName{log}\, {#1})}
\newcommand{\cLogX}[1]{\mName{log}\, {#1}}
\newcommand{\cLogB}[2]{(\mName{log}_{#1}\, {#2})}
\newcommand{\cLogBX}[2]{\mName{log}_{#1}\, {#2}}

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% THEOREM ENVIRONMENTS

\newtheorem{thm}{Theorem}[section]
\newtheorem{cor}[thm]{Corollary}
\newtheorem{lem}[thm]{Lemma}
\newtheorem{prop}[thm]{Proposition}
\newtheorem{eg}[thm]{Example}
\newtheorem{rem}[thm]{Remark}

\newtheorem{thydef}[thm]{Theory Definition}
\newtheorem{thyext}[thm]{Theory Extension}
\newtheorem{indtypethyext}[thm]{Inductive Type Theory Extension}
\newtheorem{devdef}[thm]{Development Definition}
\newtheorem{devext}[thm]{Development Extension}
\newtheorem{thytransdef}[thm]{Theory Translation Definition}
\newtheorem{thytransext}[thm]{Theory Translation Extension}
\newtheorem{devtransdef}[thm]{Development Translation Definition}
\newtheorem{devtransext}[thm]{Development Translation Extension}
\newtheorem{deftransport}[thm]{Definition Transportation}
\newtheorem{thmtransport}[thm]{Theorem Transportation}
\newtheorem{grouptransport}[thm]{Group Transportation}

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% ENVIRONMENTS

\newenvironment{theory-def}[5]
{
\color{brown!90!black}
\begin{thydef}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[] \hspace{-3ex}\textbf{Name:} #2
\item[] \hspace{-3ex}\textbf{Base types:} #3
\item[] \hspace{-3ex}\textbf{Constants:} #4
\item[] \hspace{-3ex}\textbf{Axioms:}
\end{itemize}
#5

```

```

\end{thydef}
}

\newenvironment{theory-ext}[6]
{
\color{brown!90!black}
\begin{thyext}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[] \hspace{-3ex}\textbf{Name:} #2
\item[] \hspace{-3ex}\textbf{Extends\ } #3
\item[] \hspace{-3ex}\textbf{New base types:} #4
\item[] \hspace{-3ex}\textbf{New constants:} #5
\item[] \hspace{-3ex}\textbf{New axioms:}
\end{itemize}
#6
\end{thyext}
}

\newenvironment{ind-type-theory-ext}[5]
{
\color{brown!90!black}
\begin{indtypethyext}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[] \hspace{-3ex}\textbf{Name:} #2
\item[] \hspace{-3ex}\textbf{Extends\ } #3
\item[] \hspace{-3ex}\textbf{New base type:} #4
\item[] \hspace{-3ex}\textbf{Constructors:}
\end{itemize}
#5
\end{indtypethyext}
}

\newenvironment{dev-def}[4]
{
\color{brown!90!black}
\begin{devdef}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[] \hspace{-3ex}\textbf{Name:} #2
\item[] \hspace{-3ex}\textbf{Bottom theory:} #3
\item[] \hspace{-3ex}\textbf{Definitions and theorems:}
\end{itemize}
#4

```



```

\end{devdef}
}

\newenvironment{dev-ext}[4]
{
\color{brown!90!black}
\begin{devext}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[]\hspace{-3ex}\textbf{Name:} #2
\item[]\hspace{-3ex}\textbf{Extends\ } #3
\item[]\hspace{-3ex}\textbf{New definitions and theorems:}
\end{itemize}
#4
\end{devext}
}

\newenvironment{theory-trans-def}[6]
{
\color{brown!90!black}
\begin{thytransdef}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[]\hspace{-3ex}\textbf{Name:} #2
\item[]\hspace{-3ex}\textbf{Source theory:} #3
\item[]\hspace{-3ex}\textbf{Target theory:} #4
\item[]\hspace{-3ex}\textbf{Base type mapping:}
\end{itemize}
#5
\begin{itemize}
\item[]\hspace{-3ex}\textbf{Constant mapping:}
\end{itemize}
#6
\end{thytransdef}
}

\newenvironment{theory-trans-ext}[7]
{
\color{brown!90!black}
\begin{thytransext}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[]\hspace{-3ex}\textbf{Name:} #2
\item[]\hspace{-3ex}\textbf{Extends\ } #3
\item[]\hspace{-3ex}\textbf{New source theory:} #4

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```

\item[]\hspace{-3ex}\textbf{New target theory:} #5
\item[]\hspace{-3ex}\textbf{New base type mapping:}
\end{itemize}
#6
\begin{itemize}
\item[]\hspace{-3ex}\textbf{New constant mapping:}
\end{itemize}
#7
\end{thytransect}
}

\newenvironment{dev-trans-def}[6]
{
\color{brown!90!black}
\begin{devtransdef}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[]\hspace{-3ex}\textbf{Name:} #2
\item[]\hspace{-3ex}\textbf{Source development:} #3
\item[]\hspace{-3ex}\textbf{Target development:} #4
\item[]\hspace{-3ex}\textbf{Base type mapping:}
\end{itemize}
#5
\begin{itemize}
\item[]\hspace{-3ex}\textbf{Constant mapping:}
\end{itemize}
#6
\end{devtransdef}
}

\newenvironment{dev-trans-ext}[6]
{
\color{brown!90!black}
\begin{devtransect}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[]\hspace{-3ex}\textbf{Name:} #2
\item[]\hspace{-3ex}\textbf{Extends\ } #3
\item[]\hspace{-3ex}\textbf{New source development:} #4
\item[]\hspace{-3ex}\textbf{New target development:} #5
\item[]\hspace{-3ex}\textbf{New defined constant mapping:}
\end{itemize}
#6
\end{devtransect}
}

```

```

\newenvironment{def-transport}[9]
{
\color{brown!90!black}
\begin{deftransport}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[] \hspace{-3ex}\textbf{Name:} #2
\item[] \hspace{-3ex}\textbf{Source development:} #3
\item[] \hspace{-3ex}\textbf{Target development:} #4
\item[] \hspace{-3ex}\textbf{Development morphism:} #5
\item[] \hspace{-3ex}\textbf{Definition:}
\end{itemize}
#6
\begin{itemize}
\item[] \hspace{-3ex}\textbf{Transported definition:}
\end{itemize}
#7
\begin{itemize} \setlength{\itemsep}{0pt}
\item[] \hspace{-3ex}\textbf{New target development:} #8
\item[] \hspace{-3ex}\textbf{New development morphism:} #9
\end{itemize}
\end{deftransport}
}

```

```

\newenvironment{thm-transport}[8]
{
\color{brown!90!black}
\begin{thmtransport}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[] \hspace{-3ex}\textbf{Name:} #2
\item[] \hspace{-3ex}\textbf{Source development:} #3
\item[] \hspace{-3ex}\textbf{Target development:} #4
\item[] \hspace{-3ex}\textbf{Development morphism:} #5
\item[] \hspace{-3ex}\textbf{Theorem:}
\end{itemize}
#6
\begin{itemize}
\item[] \hspace{-3ex}\textbf{Transported theorem:}
\end{itemize}
#7
\begin{itemize}
\item[] \hspace{-3ex}\textbf{New target development:} #8
\end{itemize}

```

```

\end{thmtransport}
}

\newenvironment{group-transport}[9]
{
\color{brown!90!black}
\begin{grouptransport}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[] \hspace{-3ex}\textbf{Name:} #2
\item[] \hspace{-3ex}\textbf{Source development:} #3
\item[] \hspace{-3ex}\textbf{Target development:} #4
\item[] \hspace{-3ex}\textbf{Development morphism:} #5
\item[] \hspace{-3ex}\textbf{Definitions and theorems:}
\end{itemize}
#6
\begin{itemize}
\item[] \hspace{-3ex}\textbf{Transported definitions and theorems:}
\end{itemize}
#7
\begin{itemize} \setlength{\itemsep}{0pt}
\item[] \hspace{-3ex}\textbf{New target development:} #8
\item[] \hspace{-3ex}\textbf{New development morphism:} #9
\end{itemize}
\end{grouptransport}
}

```

## References

- [1] W. M. Farmer. *Simple Type Theory: A Practical Logic for Expressing and Reasoning About Mathematical Ideas*. Computer Science Foundations and Applied Logic. Birkhäuser/Springer, 2023.