Efficient Encoding of Mathematical Knowledge

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Efficient Encoding of Computational Mathematical Knowledge

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Overview

- Classical representations
- Efficiency
- Representation by programs
- Implicit representations
- Mixed representations

Classical representations

- Expression trees
- DAGs
- Examples

Efficiency

- Computational
- Programmer
- Semantic
- User

Efficiency

- Computational
  - Time & space
- Programmer
  - Expressivity of language
  - Richness of library

Efficiency

- Semantic
  - Richness of encoding
- User
  - Usability!
  - Usefulness

Efficiency

- Primarily concerned with
  - Requirements for semantic efficiency
  - Effects of lack of semantic efficiency

Representation by programs

- Examples…

Representation by programs

- Kolmogorov complexity!
- Further (selected) examples
  - Gaia/combstruct
  - polyGCD (black box)
  - Linbox (black box)
  - Kronecker & Projective Noether

Implicit representations

- π
- RootOf(_Z^5-3*_Z+1,1.2)
- exp(z)

Implicit representations

- BesselJ(1,z)
  - Solution of 2nd order LODE
  - Enough information to generate:
    - derivative
    - series
    - floating point evaluation
    - continuity checking
    - some integration
    - and other codes!

Implicit representations

- Software architecture benefit
  - Code for ~50 functions for ~10 pieces of functionality can be generated automatically
  - Code for unnamed functions too

Implicit representations

- Defn: “solution to this problem”

Mixed representations

- 1 example

Conclusion

- Non-standard representations for knowledge promise (and often deliver) large gains in "efficiency"