An Approach to Mathematical Notation Selection

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(Demo presentation proposal)

1. Introduction

We present a software tool to select notations to be used in mathematical applications. These applications include MathML–based tools such as browsers as well as computer algebra systems supporting conversion to different math formats. Our Notation Selection Tool addresses two problems: first, it allows a user to choose which of several *different mathematical notations* to use *for the same concept*. Second, it allows disambiguation where the *same notation* could be used *for different concepts*.

There may be several notations for one concept for a number of different reasons: The *mathematical context* can lead to the same expressions being written in different ways, e.g. an ordinary derivative can be denoted as f', f_x, Df, df/dx or in some other fashion. The *area of application* may imply default notations, e.g. i for $\sqrt{-1}$ in complex analysis vs. j for the same quantity in electrical engineering. Likewise one commonly writes integrals as $\int f(x)\mathrm{d}x$, but in physics the notation $\int \mathrm{d}x\, f(x)$ is often preferred. *National and cultural* conventions are sometimes different, e.g. the tangent function is presented by *tan* in England but *tg* in Russia and China. The open interval denoted (a, b) in the United States, would be denoted as $]a, b[$ in France. The *historical period* also leads to different notations, e.g. $3\overline{a+b}$ versus the modern $3(a+b)$. The *level of mathematical sophistication* may influence the preferred representation of expressions, e.g. $a \div b$ vs. $b\overline{)a}$ vs. $\frac{a}{b}$ vs. a/b. Usually first two notations would mean exact division and would be used, e.g., in primary school.

Equally well, there are often situations where the same notation is used to represent completely different mathematical ideas. For example, the expression *lg* can mean log_{10} or log_2. The notation u' may mean "derivative", "minute", "logical not", "group inverse", "transformation performed on an original u" etc. Often the meaning is clear from context, but if several domains of mathematics are used together, then alternative notation must be used.

Our Notation Selection Tool is designed to perform conversion of mathematical expressions in XML format. The simplest use presents a graphical user interface to generate an XSLT stylesheet. This stylesheet is then used to transform conceptually–oriented Content MathML to the notationally–oriented Presentation MathML.

The interface allows the user to select notational conventions from concepts, organized by mathematical area. (See figure 1). It also allows the user to specify various file names for associated stylesheets, input and output files, browser to view conversion results, etc.

2. Implementation

Our Notation Selection Tool is written primarily in Java and uses the Swing library. The program is initialized by an XML-format specification file containing a database of concepts and alternative notations as well as template transformation rules to be applied for the selected notations. (See figure 2). The configuration bundle also includes basic XSLT stylesheet and library of images.

The notations are organized in categories, called *catalogs*, related to various areas of mathematics, e.g. arithmetic, calculus, linear algebra, combinatorics, etc. Catalogs consist of items, representing different math operations. For example catalog `ARITHMETIC` can contains items `division`, `multiplication` and `continual fractions`, catalog `CALCULUS` is subdivided into `differentiation`, `partial differentiation`, `definite` and `indefinite integration`. Each item has assigned to it list of notations choices. For example `partial differentiation` can have following notation choices: f_x, f_x', $\partial_x f$, $\nabla_x f$, $\frac{\partial f}{\partial x}$, $\mathrm{D}_x f$. Each choice defines the appearance of the notation (given as a reference to an image file), a key value to serve as a reference ID to this particular choice of notation, and an XSLT template to define the transformation of mathematical content to its presentation, according to the notation choice.

We would like to emphasize, that the actual content of catalogs, items and notations can be extended or redefined by the user of the Notation Selection Tool; the user may wish to re-use an existing notation configuration file, extend it or write another.

3. Conclusion

Advantages of this approach include flexibility and extensibility. The idea of using an initialization file for the Notation Selection Tool allows the user to *introduce new notations for existing math concepts* simply by updating this initialization file. In the same way *new mathematical concepts can be introduced* in existing settings. This entails introducing notational choices, backed by stylesheet tools to act as targets of those choices, e.g. binomial or continued fractions are defined neither in Content MathML, nor in Presentation MathML, but they can be introduced as additional stylesheet templates. The same approach allows to set preferred rendering for OpenMath CDs

This tool can be used to drive the conversion between a number of mathematical data formats, as shown in figure 3. The common characteristics of these conversions is that they typically take objects from hight–level semantic views to lower-level renderings.

A second area of possible application is that of mathematical education where students require a high degree of notational consistency within a syllabus. Our tool allows an instructor to re-use material with different notational conventions from one course to another. In distance learning students might prefer to see mathematical expressions in the format of their locality, so our tool could be used to select these preferences.

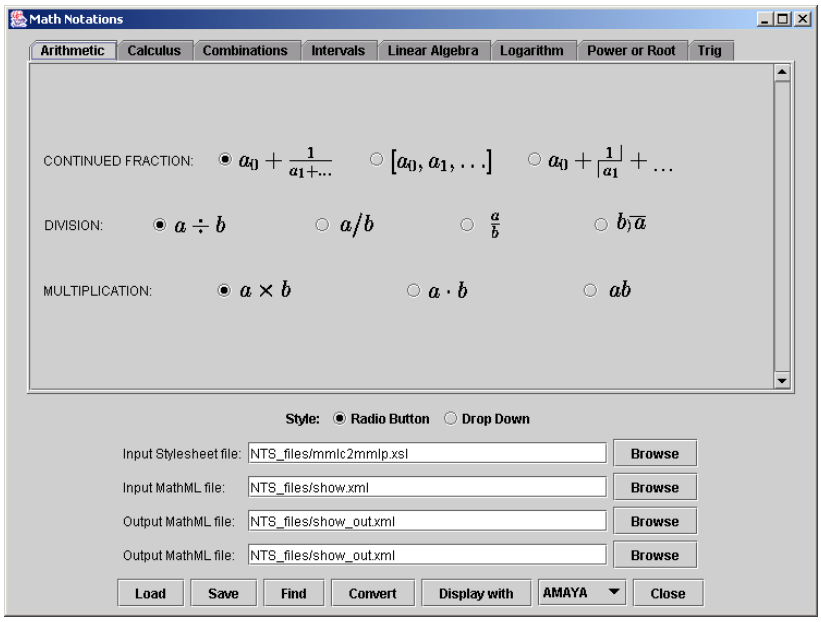

Figure 1.

```
<catalog>
  <name>Arithmetic</name>
  <itemlist>
    <item>
      <keyword> DIVISION </keyword>
      <choicelist>
        <choice>
          <image src = "div1.gif"/>
          <keyvalue> 1 </keyvalue>
          <presentation>
             ... <!-- XSLT template for this notation-->
          </presentation>
        </choice>
          ... <!-- other choices for DIVISION -->
      </choicelist>
    </item>
      ... <!-- other items for Arithmetic -->
  </itemlist>
</catalog>
```

Figure 2.

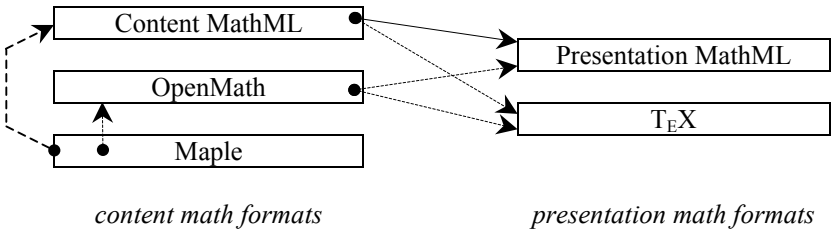

Figure 3.

References

[1] B. Naylor and S. Watt, Meta-stylesheets for the conversion of mathematical documents into multiple forms, in: Annals of Mathematics and Artificial Intelligence 38, (2003).

[2] D. Lui, A notation Selection Tools for MathML stylesheets, MSc Project University of **W**estern Ontario (2001).

[3] S. Huerter, I. Rodionov, and S. Watt, Content-Faithful Transformations for MathML, in: *MathML International Conference 2002*, http://www.mathmlconference.org/2002/presentations/huerter

[4] MathML spec: http://www.w3.org/TR/MathML2/

[5] XSLT spec: http://www.w3.org/TR/xslt.

[6] XML spec: http://www.w3.org/XML.

[7] OpenMath: www.openmath.org.