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An Approach to Mathematical Notation Selection

* Demo presentation *

Simple test:

How do we write the following?

1. the open interval with end points a and b:

2. the base 10 logarithm of x:

3. the arctangent of 2:

4. the inner product of p and q:

Simple test:

How do we write the following?

1. the open interval with end points a and b: $]a, b[$
2. the base 10 logarithm of x: $\lg x$
3. the arctangent of 2: $\mathrm{tg}^{-1} 2$
4. the inner product of p and q: $p \llcorner q$

- # Problem:

  To allow choice of mathematical notation for semantic mathematical objects

- # What is to present

  A software tool to select notations to be used in mathematical applications (such as MathML and OpenMath rendering tools, computer algebra systems etc.)

- # Two reasons why we need it

1. To choose which of several *different mathematical notations* to use *for the same concept*.
2. To disambiguate where the *same notation* could be used *for different concepts*.

• Problem overview (1)

1. Some reasons for **several notations for one concept**

- *Mathematical context*

  e.g. an ordinary derivative can be denoted as f', f_x, Df, $\frac{df}{dx}$ etc.

- *Area of application*

  e.g. i for $\sqrt{-1}$ (complex analysis) vs. j (electrical engineering).

  The integral $\int f(x)\mathrm{d}x$ (pure math) vs. $\int \mathrm{d}x\, f(x)$ (physics)

- *National and cultural conventions*

  e.g. the tangent function: *tan* (England, Canada) vs. *tg* (Russia, China);

  the open interval: (a, b) (United States) vs.]a, b[(France).

- *Historical period*

  e.g. ancient $3\overline{a+b}$ vs. the modern $3(a+b)$.

- *Level of mathematical sophistication*

  e.g. $a \div b$ vs. $b\overline{)a}$ vs. $\frac{a}{b}$ vs. a/b vs. $a\!\!\diagup\!\!_{b}$.

• Problem overview (2)

Equally well, there are often situations where the **same notation is used to represent completely different mathematical ideas**.

For example:

- *lg* can mean log_{10} or log_2;
- tg^{-1} may mean *cotangent* or *arctangent*;
- the notation *u′* may mean "*derivative*", "*minute*", "*logical not*", "*group inverse*", "*transformation performed on an original u*" etc.

Often the meaning is clear from context, but if several domains of mathematics are used together, then alternative notation must be used.

• What it does

Our Notation Selection Tool is designed to perform conversion of mathematical expressions in XML format.

The simplest use presents a graphical user interface to generate an XSLT stylesheet, which is then used to transform conceptually–oriented Content MathML to the notationally–oriented Presentation MathML.

The interface allows the user to select notational conventions from concepts, organized by mathematical area.

It also allows the user to specify various file names for associated stylesheets, input and output files, browser to view conversion results, etc.

• How it works

Our Notation Selection Tool is initialized by an XML-format *configuration file*. Its purpose is to store a database of concepts and alternative notations as well as template transformation rules to be applied for the selected notations.

```
<catalog>
  <name>Arithmetic</name>
  <itemlist>
    <item>
      <keyword> DIVISION </keyword>
      <choicelist>
        <choice>
          <image src = "div1.gif"/>
          <keyvalue> 1 </keyvalue>
          <presentation>
             ... <!-- XSLT template for this notation-->
          </presentation>
        </choice>
          ... <!-- other choices for DIVISION -->
      </choicelist>
    </item>
      ... <!-- other items for Arithmetic -->
  </itemlist>
</catalog>
```

- The notations are organized in categories, called *catalogs*, related to various areas of mathematics, e.g. arithmetic, calculus, linear algebra, combinatorics, etc.

- Catalogs consist of *items*, representing different math operations, specific to the mathematical area of this catalog.

- Each item has assigned to its list of *notations choices*.
  e.g. partial differentiation can have following notation choices:
  f_x, f_x', $\partial_x f$, $\nabla_x f$, $\frac{\partial f}{\partial x}$, $\mathrm{D}_x f$.

- Each choice defines the appearance of the notation (given as a reference to an image file), a key value to serve as a reference ID to this particular choice of notation, and an XSLT template to define the transformation of mathematical content to its presentation, according to the notation choice.

Important: the actual content of catalogs, items and notations can be extended or redefined by the user of the Notation Selection Tool:

the user may wish to re-use an existing notation configuration file, extend it or write another.

• Why it is nice

Advantages of this approach: *flexibility* and *extensibility*.

- *To introduce new notations for existing math concepts*

  the user needs simply update Notation Selection Tool initialization file.

- *New mathematical concepts can be introduced* in existing settings.

  e.g. binomial or continued fractions are defined neither in Content MathML, nor in Presentation MathML, but they can be introduced as additional stylesheet templates.

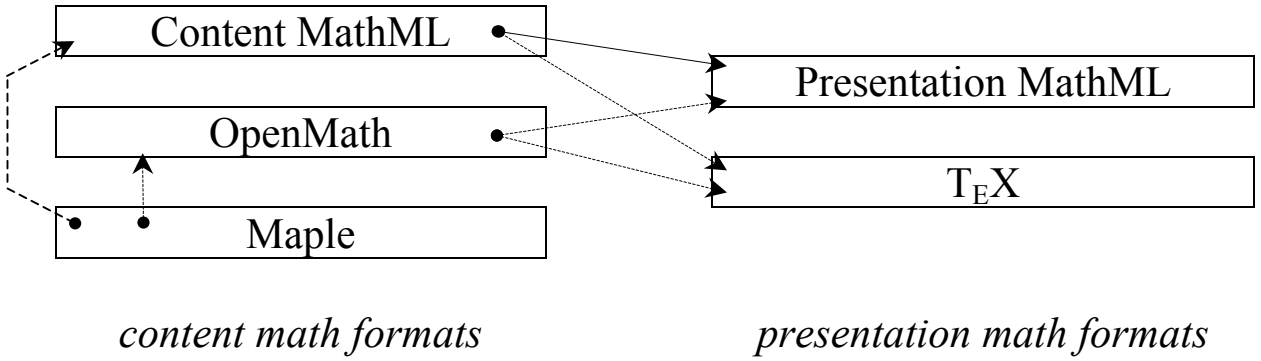
  The same approach allows to set preferred rendering for OpenMath CDs

• Where to use it

Current and possible applications:

- This tool can be used to drive the conversion between a number of mathematical data formats*.

Ex:



* The common characteristics of these conversions is that they typically take objects from hight–level semantic views to lower-level renderings.

Current and possible applications (2)

- Mathematical education:
  - Since students usually require a high degree of notational consistency within a syllabus, the Notation Selection Tool may help an instructor to re-use material with different notational conventions from one course to another.
  - In distance learning students might prefer to see mathematical expressions in the format of their locality, so the Notation Selection Tool could be used to select these preferences.

Related projects at ORCCA

www.orcca.on.ca/research.html

Software for mathematical communication and math format conversions:

- MathML ↔ LaTeX
- Content MathML → Presentation MathML
- OpenMath ↔ Content MathML
- Maple ↔ OpenMath

• References

[1] B. Naylor and S. Watt, Meta-stylesheets for the conversion of mathematical documents into multiple forms, in: Annals of Mathematics and Artificial Intelligence 38, (2003).

[2] D. Liu, A notation Selection Tools for MathML stylesheets, MSc Project University of Western Ontario (2001).

[3] S. Huerter, I. Rodionov, and S. Watt, Content-Faithful Transformations for MathML, in: *MathML International Conference 2002*, http://www.mathmlconference.org/2002/presentations/huerter

[4] MathML spec: http://www.w3.org/TR/MathML2/

[5] XSLT spec: http://www.w3.org/TR/xslt.

[6] XML spec: http://www.w3.org/XML.

[7] OpenMath: www.openmath.org.