# Corrigenda to Simple Type Theory

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October 25, 2024

#### Abstract

This document presents corrections to errors in the textbook Simple Type Theory: A Practical Logic for Expressing and Reasoning About Mathematical Ideas.

#### 1 Introduction

The textbook Simple Type Theory: A Practical Logic for Expressing and Reasoning About Mathematical Ideas [3] is an introduction to simple type theory [2]. It presents a practice-oriented logic called Alonzo that is based on Alonzo Church's formulation of simple type theory known as Church's type theory [1]. Unlike traditional predicate logics, Alonzo admits undefined expressions. The book illustrates using Alonzo how simple type theory is exceptionally well suited for expressing and reasoning about mathematical ideas.

We have found 18 minor errors in *Simple Type Theory* that the reader may not notice or know how to correct. This document describes the errors and presents corrections for them. There are other errors in *Simple Type Theory*, mostly of a typographical nature, that the reader should be able to immediately identify and correct.

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On p. 27 (Section 3.4), the definition of a finite sequence as a partial function on the natural numbers should be written as:

A finite sequence of values in A can be formalized as a function  $s : \mathbb{N} \to A$  such that, for some  $n \in \mathbb{N}$ , s(m) is defined iff m < n.

That is, "m < n" should be used instead of " $m \le n$ ".

The same mistake appears on p. 127 (Section 10.0) in the definition of a finite sequence and on p. 129 (Section 10.2) in the notational definition for  $lists_{\{\alpha \to \beta\}}$  given in Table 10.1. And on p. 151 (Subsection 12.3.5) in Def27, " $m \leq n$ " needs to be changed to "m < n".

# Corrigendum 2

The definition of a type on p. 38 (Section 4.3) confuses "type" and "set of types". This is corrected by beginning the definition with:

A *type* of Alonzo is a string of symbols defined inductively by the following formation rules:

Similarly, the definition of a expression on p. 40 (Section 4.4) confuses "expression" and "set of expressions". This is corrected by beginning the definition with:

An expression of type  $\alpha$  of Alonzo is a string of symbols defined inductively by the following formation rules:

The same mistake appears on pp. 200 and 201 (Subsection 14.2.2) in the definitions of a sort and expression of AlonzoS.

#### Corrigendum 3

On p. 63 (Section 5.11), Exercise 5.11.7 asks the reader to prove that  $(\mathbb{M}, \leq)$  is a meet-semilattice, which it is not. Instead the exercise should ask the reader to prove that  $(\mathbb{M}, \leq)$  is a weak partial order with bottom element.

On p. 65 (Section 6.1), the notation  $(\mathbf{A}_o \mapsto \mathbf{B}_\alpha \mid \mathbf{C}_\alpha)$  for a conditional expression is defined in Table 6.1 as the application of the pseudoconstant if  $_{o\to\alpha\to\alpha\to\alpha}$ . As a result,  $V_{\varphi}^M((\mathbf{A}_o\mapsto\mathbf{B}_\alpha\mid\mathbf{C}_\alpha))$  is undefined when  $V_{\varphi}^M(\mathbf{A}_o) = T$ ,  $V_{\varphi}^M(\mathbf{B}_\alpha)$  is defined, and  $V_{\varphi}^M(\mathbf{C}_\alpha)$  is undefined. Instead,  $V_{\varphi}^M((\mathbf{A}_o\mapsto\mathbf{B}_\alpha\mid\mathbf{C}_\alpha))$  should equal  $V_{\varphi}^M(\mathbf{B}_\alpha)$  in this case. Therefore, a conditional expression must be defined using an abbreviation instead of an application of a pseudoconstant.

The last two notational definitions in Table 6.1 need to be removed, and the following three notational definitions need to be added to the end of Table 6.4:

$$\begin{array}{ll} \mathsf{IF}(\mathbf{A}_{o},\mathbf{B}_{o},\mathbf{C}_{o}) & \text{stands for} & (\mathbf{A}_{o}\Rightarrow\mathbf{B}_{o})\wedge(\neg\mathbf{A}_{o}\Rightarrow\mathbf{C}_{o}). \\ \mathsf{IF}(\mathbf{A}_{o},\mathbf{B}_{\alpha},\mathbf{C}_{\alpha}) & \text{stands for} & \mathsf{I}\,x:\alpha \ & (\mathbf{A}_{o}\Rightarrow x=\mathbf{B}_{\alpha})\wedge(\neg\mathbf{A}_{o}\Rightarrow x=\mathbf{C}_{\alpha}) \\ & & \text{where } \alpha\neq o. \\ (\mathbf{A}_{o}\mapsto\mathbf{B}_{\alpha}\mid\mathbf{C}_{\alpha}) & \text{stands for} & \mathsf{IF}(\mathbf{A}_{o},\mathbf{B}_{\alpha},\mathbf{C}_{\alpha}) \end{array}$$

Sections 6.1, 6.4, and 6.8 must be modified to accommodate this change. Also, on p. 203 (Subsection 14.2.3), the notational definitions for the pseudoconstant if  $_{o\to\sigma\to\tau\to\alpha}$  and the conditional expression ( $\mathbf{A}_o \mapsto \mathbf{B}_\sigma \mid \mathbf{C}_\tau$ ) should be removed from Table 14.3.

#### Corrigendum 5

On p. 67 (Section 6.1), the proof of Theorem 5.9 (every finite general model is a standard model) is incomplete. We must show that, if M is a finite general model of L, then  $D^{M}_{\alpha\to\beta}$  is full for all  $\alpha, \beta \in \mathcal{T}(L)$ . There are two cases: (1)  $\alpha, \beta \in \mathcal{T}(L)$  with  $\beta = o$  and (2)  $\alpha, \beta \in \mathcal{T}(L)$  with  $\beta \neq o$ . Only the second case is proved; the proof of the first case is missing. However, the proof of first case is similar to, but simpler than, the proof of the second case.

#### **Corrigendum 6**

On p. 68 (Section 6.2), the condition for the notational definition for

 $(\exists ! \mathbf{x} : \alpha \cdot \mathbf{A}_o)$ 

given in Table 6.3 should be

where y does not occur in  $(\lambda \mathbf{x} : \alpha \cdot \mathbf{A}_o)$ 

instead of

where y is not free in  $(\lambda \mathbf{x} : \alpha \cdot \mathbf{A}_o)$ .

# Corrigendum 7

On p. 74 (Section 6.9), the notational definitions for  $(\mathbf{Q}_{\{\alpha\}} \to \mathbf{R}_{\{\beta\}})$ ,  $(\alpha \to \mathbf{R}_{\{\beta\}})$ , and  $(\mathbf{Q}_{\{\alpha\}} \to \beta)$  given in Table 6.9 require the condition

where  $\beta \neq o$ .

To complete this correction, the following notational definitions need to be added to Table 6.9:

$(\mathbf{Q}_{\{\alpha\}} \to o)$	stands for	$\{s: \{\alpha\} \mid s \subseteq \mathbf{Q}_{\{\alpha\}}\}.$
$\mathcal{P}(\mathbf{Q}_{\{lpha\}})$	stands for	$\mathbf{Q}_{\{\alpha\}} \to o.$

# **Corrigendum 8**

The introductory remarks in Example 9.6 on p. 98 (Section 9.1) say

... Giuseppe Peano presented a characterization of the natural numbers based on 0 and the successor function S. We express his characterization in Alonzo as the following theory PA called *Peano Arithmetic*:

However, this is not true: Peano's characterization of the natural numbers starts with 1, not 0. Thus the introductory remarks should instead say

... Giuseppe Peano presented a characterization of the natural numbers based on 1 and the successor function S. We express his characterization in Alonzo as the following theory PA (in which we start with 0 instead of 1) called *Peano Arithmetic*:

Example 4.47 on p. 114 and the proof of Theorem 9.49 on p. 115 (both in Section 9.4) assume, without proof, that there is a standard model of RAT. This hole in the example can be eliminated by simplifying RAT and then reorganizing the proof that RAT is categorical in the standard sense as follows:

**Example 9.47 (Rational Numbers Order)** In Example 9.18, we defined DWTOWE, a theory of dense weak total orders without endpoints. Let us now define an extension RAT of DWTOWE called *Rational Numbers Order*.

Theory Extension 9.48 (Rational Numbers Order)

```
Name: RAT
Extends DWTOWE
New base types: (none)
New constants: (none)
New axioms:
```

9. COUNT $(U_{\{S\}})$ .

(S is countable.)

All dense weak total orders without endpoints are infinite and either countable, such as  $(\mathbb{Q}, \leq)$ , or uncountable, such as  $(\mathbb{R}, \leq)$ . Let M be a standard model of DWTOWE. Then, by virtue of axiom 9, M is model of RAT iff  $D_S^M$  is countable. Hence the standard model  $M_{\text{rat}}$  of DWTOWE that defines  $(\mathbb{Q}, \leq)$  is a model of RAT.

**Proposition 9.49** RAT is a specification in the standard sense of all countable dense weak total orders without endpoints.

**Proof** This follows from the fact given in Example 9.47 that a standard model M of DWTOWE is a model of RAT iff  $D_S^M$  is countable.

**Theorem 9.50 (Categoricity of RAT) RAT** is categorical in the standard sense.

**Proof** As shown in Example 9.47, the standard model  $M_{rat}$  of DWTOWE that defines  $(\mathbb{Q}, \leq)$  is a model of RAT. Georg Cantor proved in 1895 that every two countable dense weak total orders without endpoints are order isomorphic [20]. Thus, by Proposition 9.49, every model of RAT is isomorphic to  $M_{rat}$ , and so RAT is categorical in the standard sense.

On p. 115 (Section 9.5), a theory  $T = (L, \Gamma)$  is defined to be *(semantically)* complete if either  $T \models \mathbf{A}_o$  or  $T \models \neg \mathbf{A}_o$  holds for all sentences  $\mathbf{A}_o$  of L. Hence every unsatisfiable theory is incomplete. Therefore, Example 9.51 (Theory of False) and Proposition 9.52 are incorrect and should be removed.

# Corrigendum 11

On p. 117 (Section 9.5), Example 9.60 (Theory of a Countably Infinite Domain) should be removed since the theory CID defined in the example is not complete as asserted.

### Corrigendum 12

On p. 126 (Section 9.9), the hint for Exercise 9.9.17 is misleading and should be removed.

#### Corrigendum 13

On p. 131 (Section 11.1), the definition of a theorem package P being valid in a theory should be:

*P* is valid in a theory  $T = (L, \Gamma)$  if  $\mathbf{A}_o$  is a sentence of *L* and  $\pi$  is a proof of  $\mathbf{A}_o$  from  $\Gamma$ .

That is,  $\mathbf{A}_o$  is required to be a sentence of L.

#### Corrigendum 14

On p. 164 (Subsection 13.3.2), the statement and proof of Lemma 13.9 tacitly assume that  $\overline{\mu}(\alpha)$  is defined, but  $\overline{\mu}(\alpha)$  may be undefined in some cases. The lemma and its proof are corrected as follows:

**Lemma 13.9** Let  $\Phi = (\mu, \nu)$  be a translation from  $T_1$  to  $T_2$ .

- 1. If  $\alpha \in \mathcal{T}(L_1)$  and  $\overline{\mu}(\alpha) \in \mathcal{Q}_2$ , then  $T_2 \vDash \overline{\mu}(\alpha) \downarrow \Rightarrow \overline{\nu}(U_{\{\alpha\}}) = \overline{\mu}(\alpha)$ .
- 2. If  $\alpha \in \mathcal{T}(L_1)$  and  $\overline{\mu}(\alpha) \in \mathcal{Q}_2$ , then  $T_2 \vDash \overline{\mu}(\alpha) \uparrow \Rightarrow \overline{\nu}(U_{\{\alpha\}}) = \emptyset_{\{\tau(\overline{\mu}(\alpha))\}}$ .

3. If  $\mathbf{a} \in \mathcal{B}_1$  and  $\mu(\mathbf{a}) \in \mathcal{Q}_2$ , then

$$T_2 \vDash \overline{\nu}(U_{\{\mathbf{a}\}} \neq \emptyset_{\{\mathbf{a}\}}) \Leftrightarrow (\mu(\mathbf{a}) \downarrow \land \mu(\mathbf{a}) \neq \emptyset_{\{\tau(\mu(\mathbf{a}))\}}).$$

- 4. If  $\mathbf{c}_{\alpha} \in \mathcal{C}_1$  and  $\overline{\mu}(\alpha) \in \mathcal{T}_2$ , then  $T_2 \vDash \overline{\nu}(\mathbf{c}_{\alpha} \downarrow U_{\{\alpha\}}) \Leftrightarrow \nu(\mathbf{c}_{\alpha}) \downarrow$ .
- 5. If  $\mathbf{c}_{\alpha} \in \mathcal{C}_1$  and  $\overline{\mu}(\alpha) \in \mathcal{Q}_2$ , then

$$T_2 \vDash \overline{\mu}(\alpha) \downarrow \Rightarrow \overline{\nu}(\mathbf{c}_\alpha \downarrow U_{\{\alpha\}}) \Leftrightarrow \nu(\mathbf{c}_\alpha) \downarrow \overline{\mu}(\alpha).$$

**Proof** Let  $\alpha \in \mathcal{T}(L_1)$ ,  $\overline{\mu}(\alpha) \in \mathcal{Q}_2$ , and  $(\star) M$  be a general model of  $T_2$  in which  $\overline{\mu}(\alpha) \downarrow$  is true. We must show that  $\overline{\nu}(U_{\{\alpha\}}) = \overline{\mu}(\alpha)$  is true in M. Then

$$\overline{\nu}(U_{\{\alpha\}})$$

$$\equiv \overline{\nu}(\lambda x : \alpha . T_o)$$

$$\equiv \lambda x : \overline{\mu}(\alpha) . T_o$$

$$\equiv \lambda x : \tau(\overline{\mu}(\alpha)) . (x \in \overline{\mu}(\alpha) \mapsto T_o \mid F_o).$$

by the definition of  $\overline{\nu}$  and notational definitions. The last expression is clearly equal to  $\lambda x : \tau(\overline{\mu}(\alpha)) \cdot \overline{\mu}(\alpha) x$ , which is equal to  $\overline{\mu}(\alpha)$  in M by  $(\star)$ . This proves part 1. Part 2 follows from the proof of part 1 and the notational definition for the empty set pseudoconstant. Part 3 follows immediately from the definition of  $\overline{\nu}$  and parts 1 and 2. Part 4 follows from the definition of  $\overline{\nu}$  and the notational definition for the defined-in-quasitype operator. And part 5 follows immediately from the definition of  $\overline{\nu}$  and part 1.  $\Box$ 

#### Corrigendum 15

On p. 166 (Subsection 13.3.2), the proof of Theorem 13.13 (Morphism Theorem) is incorrect for nonnormal translations. The extraction of the structure  $M_1$  from a model  $M_2$  of  $T_2$  is more complicated when  $\Phi$  is not a normal translation and it takes more work to prove that, if each obligation of  $\Phi$  is true in  $M_2$ , then  $M_1$  is a model of  $T_1$ .

More specifically, the text between Lemma 13.11 and Theorem 13.13 needs be replaced with the following text (which will require the subsequence theorems and examples to be renumbered):

Let  $\Phi = (\mu, \nu)$  be a translation from  $T_1$  to  $T_2$ ,  $M_2 = (\{D_{\alpha}^2 \mid \alpha \in \mathcal{T}_2\}, I_2)$  be a model of  $T_2$  in which each obligation of  $\Phi$  of the first and second kind is true, and  $\varphi \in \operatorname{assign}(M_2)$ . We will extract an interpretation  $M_1$  from  $M_2$ using  $\Phi$  as follows. To start, let us define

$$D_{\alpha} = \begin{cases} D_{\overline{\mu}(\alpha)}^{2} & \text{if } \overline{\mu}(\alpha) \in \mathcal{T}_{2} \\ \{d \in D_{\tau(\overline{\mu}(\alpha))}^{2} \mid d \in V_{\varphi}^{M_{2}}(\overline{\mu}(\alpha))\} & \text{if } \overline{\mu}(\alpha) \in \mathcal{Q}_{2} \end{cases}$$

for  $\alpha \in \mathcal{T}_1$ .

**Lemma 13.12**  $D_{\alpha}$  is nonempty for all  $\alpha \in \mathcal{T}_1$ .

**Proof** The proof is by induction on the syntactic structure of types.  $D_o$  is obviously nonempty. If  $\overline{\mu}(\mathbf{a}) \in \mathcal{T}_2$ , then  $D_{\mathbf{a}}$  is also obviously nonempty. If  $\overline{\mu}(\mathbf{a}) \in \mathcal{Q}_2$ , then  $D_{\mathbf{a}}$  is nonempty by part 3 of Lemma 13.9 and the fact that the obligations of  $\Phi$  of the first kind are true in  $M_2$ . If  $\alpha = \beta \to \gamma$  and  $D_\beta$  and  $D_\gamma$  are nonempty, then  $D_\alpha$  is nonempty since it contains the empty function. If  $\alpha = \beta \times \gamma$  and  $D_\beta$  and  $D_\gamma$  are nonempty, then  $D_\alpha$  are nonempty, then  $D_\alpha$  is nonempty since it contains the empty since it equals  $D_\beta \times D_\gamma$ .

If  $\Phi$  is normal, then  $\{D_{\alpha} \mid \alpha \in \mathcal{T}_1\}$  is clearly a frame for  $L_1$ . On the other hand, if  $\Phi$  is not normal, then  $\{D_{\alpha} \mid \alpha \in \mathcal{T}_1\}$  may not be a frame. This will happen because  $D_{\alpha \to \beta} \subseteq D^2_{\tau(\overline{\mu}(\alpha))} \to D^2_{\tau(\overline{\mu}(\beta))}$  and thus  $D_{\alpha \to \beta} \not\subseteq D_{\alpha} \to D_{\beta}$  if  $D_{\alpha} \subset D^2_{\tau(\overline{\mu}(\alpha))}$  or  $D_{\beta} \subset D^2_{\tau(\overline{\mu}(\beta))}$ . However,  $\mathsf{dom}(f) \subseteq D_{\alpha}$  and  $\mathsf{ran}(f) \subseteq D_{\beta}$ for all  $f \in D_{\alpha \to \beta}$ . Therefore, we can turn a recalcitrant  $\{D_{\alpha} \mid \alpha \in \mathcal{T}_1\}$  of this kind into a frame by modifying the members of its function domains as follows.

We will define  $D_{\alpha}^1$  and  $H_{\alpha}: D_{\alpha} \to D_{\alpha}^1$  for all  $\alpha \in \mathcal{T}_1$  by recursion on the syntactic structure of types. We will also prove that  $H_{\alpha}$  is a bijection for all  $\alpha \in \mathcal{T}_1$  by induction on the syntactic structure of types. There are four cases to consider:

**Case 1**:  $\alpha = o$ . Define  $D_{\alpha}^1 = D_{\alpha}$  and  $H_{\alpha}$  to be the identity function.  $H_{\alpha}$  is clearly a bijection.

**Case 2**:  $\alpha = \mathbf{a}$ . Define  $D_{\alpha}^1 = D_{\alpha}$  and H to be the identity function.  $H_{\alpha}$  is clearly a bijection.

**Case 3**:  $\alpha = \beta \to \gamma$ . Assume  $D_{\beta}^{1}$  and  $D_{\gamma}^{1}$  are defined and  $H_{\beta} : D_{\beta} \to D_{\beta}^{1}$ and  $H_{\gamma} : D_{\gamma} \to D_{\gamma}^{1}$  are bijections. Let  $f \in D_{\alpha}$ . Define  $H_{\alpha}(f)$  to be the function  $g : D_{\beta}^{1} \to D_{\gamma}^{1}$  such that  $g(x) \simeq H_{\gamma}(f(H_{\beta}^{-1}(x)))^{1}$  for all  $x \in D_{\beta}^{1}$ .

That is,  $g(x) = H_{\gamma}(f(H_{\beta}^{-1}(x)))$  if  $f(H_{\beta}^{-1}(x))$  is defined and g(x) is undefined otherwise.

Then define  $D^1_{\alpha} = H_{\alpha}[D_{\alpha}]$ , i.e., the image of  $D_{\alpha}$  under  $H_{\alpha}$ .  $H_{\alpha}$  is clearly a bijection since  $H_{\beta}$  and  $H_{\gamma}$  are bijections.

**Case 4**:  $\alpha = \beta \times \gamma$ . Assume  $D^1_\beta$  and  $D^1_\gamma$  are defined and  $H_\beta : D_\beta \to D^1_\beta$ and  $H_{\gamma}: D_{\gamma} \to D_{\gamma}^1$  are bijections. Define  $D_{\alpha}^1 = D_{\beta}^1 \times D_{\gamma}^1$  and  $H_{\alpha}((a, b)) =$  $(H_{\beta}(a), H_{\gamma}(b))$ .  $H_{\alpha}$  is clearly a bijection since  $H_{\beta}$  and  $H_{\gamma}$  are bijections.

**Lemma 13.13**  $\{D^1_{\alpha} \mid \alpha \in \mathcal{T}_1\}$  is a frame of  $L_1$ .

**Proof** Follows from Lemma 13.12 and the construction of the  $D^1_{\alpha}$ .

Finally, define  $I_1(\mathbf{c}_{\alpha}) \simeq H_{\alpha}(V_{\omega}^{M_2}(\nu(\mathbf{c}_{\alpha})))^2$  for  $\mathbf{c}_{\alpha} \in \mathcal{C}_1$  and

 $M_1 = (\{D^1_{\alpha} \mid \alpha \in \mathcal{T}_1\}, I_1).$ 

**Lemma 13.14**  $M_1$  is an interpretation of  $L_1$ .

**Proof**  $\{D^1_{\alpha} \mid \alpha \in \mathcal{T}_1\}$  is a frame of  $L_1$  by Lemma 13.13. Let  $\mathbf{c}_{\alpha} \in \mathcal{C}_1$ . We must show that  $I_1(\mathbf{c}_{\alpha}) \in D^1_{\alpha}$ . If  $\overline{\mu}(\alpha) \in \mathcal{T}_2$ , then  $I_1(\mathbf{c}_{\alpha}) = H_{\alpha}(V^{M_2}_{\varphi}(\nu(\mathbf{c}_{\alpha}))) \in$  $H_{\alpha}[D^2_{\overline{\mu}(\alpha)}] = H_{\alpha}[D_{\alpha}] = D^1_{\alpha}$  by part 4 of Lemma 13.9, the fact that the obligations of  $\Phi$  of the second kind are true in  $M_2$ , and  $H_\alpha : D_\alpha \to D^1_\alpha$ . If  $\overline{\mu}(\alpha) \in \mathcal{Q}_2$ , then  $I_1(\mathbf{c}_{\alpha}) = H_{\alpha}(V_{\varphi}^{M_2}(\nu(\mathbf{c}_{\alpha}))) \in H_{\alpha}[\{d \in D^2_{\tau(\overline{\mu}(\alpha))} \mid d \in U_{\varphi}^{(1)}(\mu(\alpha))\}]$  $V^{M_2}_{\varphi}(\overline{\mu}(\alpha))\} = H_{\alpha}[D_{\alpha}] = D^1_{\alpha}$  by part 5 of Lemma 13.9, the fact that the obligations of  $\Phi$  of the second kind are true in  $M_2$ , and  $H_\alpha : D_\alpha \to D^1_\alpha$ . Therefore,  $M_1$  is an interpretation of  $L_1$ .

**Lemma 13.15** Let  $\Phi = (\mu, \nu)$  be a translation from  $T_1$  to  $T_2$ ,  $M_2$  be a model of  $T_2$  in which each obligation of  $\Phi$  is true in  $M_2$ , and  $M_1$  be the structure defined above. Then  $M_1$  is a model of  $T_1$ .

**Proof**  $M_1$  is an interpretation of  $L_1$  by Lemma 13.14.

For all  $\mathbf{A}_{\alpha} \in \mathcal{E}_1$  and  $\varphi \in \operatorname{assign}(M_1)$ , define

(\*)  $V_{\varphi}^{M_1}(\mathbf{A}_{\alpha}) \simeq H_{\alpha}(V_{\overline{\nu}(\alpha)}^{M_2}(\overline{\nu}(\mathbf{A}_{\alpha})))^3,$ 

<sup>2</sup>That is,  $I_1(\mathbf{c}_{\alpha}) = H_{\alpha}(V_{\varphi}^{M_2}(\nu(\mathbf{c}_{\alpha})))$  if  $V_{\varphi}^{M_2}(\nu(\mathbf{c}_{\alpha}))$  is defined and  $I_1(\mathbf{c}_{\alpha})$  is undefined otherwise.

<sup>3</sup>That is,  $V_{\varphi}^{M_1}(\mathbf{A}_{\alpha}) = H_{\alpha}(V_{\overline{\nu}(\varphi)}^{M_2}(\overline{\nu}(\mathbf{A}_{\alpha})))$  if  $V_{\overline{\nu}(\varphi)}^{M_2}(\overline{\nu}(\mathbf{A}_{\alpha}))$  is defined and  $V_{\varphi}^{M_1}(\mathbf{A}_{\alpha})$  is undefined otherwise.

where  $\overline{\nu}(\varphi)$  is any  $\psi \in \operatorname{assign}(M_2)$  such that  $\psi(\overline{\nu}((\mathbf{x}:\beta))) = H_{\beta}^{-1}(\varphi((\mathbf{x}:\beta)))$ for all variables  $(\mathbf{x}:\beta) \in \mathcal{E}_1$ . By induction on the syntactic structure of expressions,  $V_{\varphi}^{M_1}$  satisfies the seven conditions of the definition of a general model. Therefore,  $M_1$  is a general model of  $L_1$ .

 $(\star)$  implies

(\*\*)  $M_1 \vDash \mathbf{A}_o$  iff  $M_2 \vDash \overline{\nu}(\mathbf{A}_o)$  for all sentences  $\mathbf{A}_o \in \mathcal{E}_1$ .

If  $\mathbf{A}_o \in \Gamma_1$ , then  $M_2 \models \overline{\nu}(\mathbf{A}_o)$  since each obligation of  $\Phi$  of the third kind is true in  $M_2$ . Hence  $M_1 \models \mathbf{A}_o$  for all  $\mathbf{A}_o \in \Gamma_1$  by (\*\*). Therefore,  $M_1$  is a model of  $T_1$ .

#### Corrigendum 16

The definition transportation and theorem transportation modules defined on p. 184 (Subsection 13.4.2) should include *Source development* and *Target development* fields.

### Corrigendum 17

The F2 component of the definition of a frame for AlonzoS on p. 202 (Subsection 14.2.3) is not correctly formulated since f(d) = F for all  $f \in D_{\sigma \to o}$ and  $d \in D_{\overline{\xi}(\sigma)} \setminus D_{\sigma}$ . The following is the correct formulation:

F2. Predicate domain:  $D_{\sigma \to o}$  is a set of some total functions f from  $D_{\overline{\xi}(\sigma)}$  to  $D_o$  such that f(d) = F for all  $d \in D_{\overline{\xi}(\sigma)} \setminus D_{\sigma}$  for  $\sigma \in \mathcal{U}(L)$ .

#### Corrigendum 18

The V9 component of the definition of a general model of AlonzoQE on p. 208 (Subsection 14.3.3) is not correct when  $V_{\varphi}^{M}(\mathbf{A}_{\epsilon})$  denotes an expression  $\mathbf{E}_{\alpha}$  such that  $V_{\varphi}^{M}(\mathbf{E}_{\alpha})$  is undefined. The following is the correct formulation:

V9.  $V_{\varphi}^{M}(\llbracket \mathbf{A}_{\epsilon} \rrbracket_{\alpha}) = V_{\varphi}^{M}(V_{\varphi}^{M}(\mathbf{A}_{\epsilon}))$  if  $V_{\varphi}^{M}(\mathbf{A}_{\epsilon})$  is an expression  $\mathbf{E}_{\alpha}$  such that  $V_{\varphi}^{M}(\mathbf{E}_{\alpha})$  is defined. Otherwise,  $V_{\varphi}^{M}(\llbracket \mathbf{A}_{\epsilon} \rrbracket_{\alpha}) = \mathbf{F}$  if  $\alpha = o$  and is undefined if  $\alpha \neq o$ .

To complete this correction, Theorem 14.9 and its proof must be changed as follows where several = symbols are replaced with  $\simeq$  symbols: **Theorem 14.9 (Law of Disquotation)** Let Alonzo QE be the logic. Then  $\llbracket \mathbf{A}_{\alpha} \urcorner \rrbracket_{\alpha} \simeq \mathbf{A}_{\alpha}$  is valid.

**Proof** Let  $\mathbf{A}_{\alpha}$  be an eval-free expression of a language L, M be a general model of L, and  $\varphi \in \operatorname{assign}(M)$ . (a)  $V_{\varphi}^{M}(\ulcorner \mathbf{A}_{\alpha} \urcorner) = \mathbf{A}_{\alpha}$  by condition V8 of the definition of a general model. (a) implies (b)  $V_{\varphi}^{M}(\ulcorner \mathbf{A}_{\alpha} \urcorner)$  is an expression of type  $\alpha$ .

Assume  $V_{\varphi}^{M}(\mathbf{A}_{\alpha})$  is defined. Then

$$V_{\varphi}^{M}(\llbracket \mathbf{A}_{\alpha} \mathsf{T} \rrbracket_{\alpha}) = V_{\varphi}^{M}(V_{\varphi}^{M}(\mathbf{A}_{\alpha} \mathsf{T}))$$
(1)

$$=V_{\varphi}^{M}(\mathbf{A}_{\alpha}). \tag{2}$$

(1) is by (a), (b), and condition V9 of the definition of a general model; and (2) is by (a). Now assume  $V_{\varphi}^{M}(\mathbf{A}_{\alpha})$  is undefined. Then  $V_{\varphi}^{M}(\llbracket \mathbf{A}_{\alpha} \urcorner \rrbracket_{\alpha})$  is undefined by (a) and condition V9, and so  $V_{\varphi}^{M}(\llbracket \mathbf{A}_{\alpha} \urcorner \rrbracket_{\alpha}) \simeq V_{\varphi}^{M}(\mathbf{A}_{\alpha})$ .

undefined by (a) and condition V9, and so  $V_{\varphi}^{M}(\llbracket [ \mathbf{A}_{\alpha} ] ]_{\alpha}) \simeq V_{\varphi}^{M}(\mathbf{A}_{\alpha}).$ Hence  $V_{\varphi}^{M}(\llbracket [ \mathbf{A}_{\alpha} ] ]_{\alpha}) \simeq V_{\varphi}^{M}(\mathbf{A}_{\alpha})$  for all general models M that interpret  $\mathbf{A}_{\alpha}$  and all  $\varphi \in \operatorname{assign}(M)$ , and so  $\llbracket [ \mathbf{A}_{\alpha} ] ]_{\alpha} \simeq \mathbf{A}_{\alpha}$  is valid by part 5 of Lemma 6.5.

#### References

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