Corrigenda to *Simple Type Theory*

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Abstract

This document presents corrections to errors in the textbook *Simple Type Theory: A Practical Logic for Expressing and Reasoning About Mathematical Ideas.*

1 Introduction

The textbook *Simple Type Theory: A Practical Logic for Expressing and Reasoning About Mathematical Ideas* [3] is an introduction to *simple type theory* [2]. It presents a practice-oriented logic called *Alonzo* that is based on Alonzo Church's formulation of simple type theory known as *Church's type theory* [1]. Unlike traditional predicate logics, Alonzo admits undefined expressions. The book illustrates using Alonzo how simple type theory is exceptionally well suited for expressing and reasoning about mathematical ideas.

We have found 16 minor errors in *Simple Type Theory* that the reader may not notice or know how to correct. This document describes the errors and presents corrections for them. There are other errors in *Simple Type Theory*, mostly of a typographical nature, that the reader should be able to immediately identify and correct.

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Corrigendum 1

On p. 27 (Section 3.4), the definition of a finite sequence as a partial function on the natural numbers should be written as:

> A finite sequence of values in A can be formalized as a function $s : \mathbb{N} \rightarrow A$ such that, for some $n \in \mathbb{N}$, $s(m)$ is defined iff $m < n$.

That is, "$m < n$" should be used instead of "$m \leq n$".

The same mistake appears on p. 127 (Section 10.0) in the definition of a finite sequence and on p. 129 (Section 10.2) in the notational definition for $\mathsf{lists}_{\{\alpha \rightarrow \beta\}}$ given in Table 10.1. And on p. 151 (Subsection 12.3.5) in Def27, "$m \leq n$" needs to be changed to "$m < n$".

Corrigendum 2

The definition of a type on p. 38 (Section 4.3) confuses "type" and "set of types". This is corrected by beginning the definition with:

> A *type* of Alonzo is a string of symbols defined inductively by the following formation rules:

Similarly, the definition of a expression on p. 40 (Section 4.4) confuses "expression" and "set of expressions". This is corrected by beginning the definition with:

> An *expression of type* α of Alonzo is a string of symbols defined inductively by the following formation rules:

The same mistake appears on pp. 200 and 201 (Subsection 14.2.2) in the definitions of a sort and expression of AlonzoS.

Corrigendum 3

On p. 63 (Section 5.11), Exercise 5.11.7 asks the reader to prove that $(\mathbb{M}, \leq)$ is a meet-semilattice, which it is not. Instead the exercise should ask the reader to prove that $(\mathbb{M}, \leq)$ is a weak partial order with bottom element.

Corrigendum 4

On p. 65 (Section 6.1), the notation $(\mathbf{A}_o \mapsto \mathbf{B}_\alpha \mid \mathbf{C}_\alpha)$ for a conditional expression is defined in Table 6.1 as the application of the pseudoconstant $\mathsf{if}_{o \to \alpha \to \alpha \to \alpha}$. As a result, $V_\varphi^M((\mathbf{A}_o \mapsto \mathbf{B}_\alpha \mid \mathbf{C}_\alpha))$ is undefined when $V_\varphi^M(\mathbf{A}_o) = \mathrm{T}$, $V_\varphi^M(\mathbf{B}_\alpha)$ is defined, and $V_\varphi^M(\mathbf{C}_\alpha)$ is undefined. Instead, $V_\varphi^M((\mathbf{A}_o \mapsto \mathbf{B}_\alpha \mid \mathbf{C}_\alpha))$ should equal $V_\varphi^M(\mathbf{B}_\alpha)$ in this case. Therefore, a conditional expression must be defined using an abbreviation instead of an application of a pseudoconstant.

The last two notational definitions in Table 6.1 need to be removed, and the following three notational definitions need to be added to the end of Table 6.4:

$\mathsf{IF}(\mathbf{A}_o, \mathbf{B}_o, \mathbf{C}_o)$	stands for	$(\mathbf{A}_o \Rightarrow \mathbf{B}_o) \wedge (\neg \mathbf{A}_o \Rightarrow \mathbf{C}_o)$.
$\mathsf{IF}(\mathbf{A}_o, \mathbf{B}_\alpha, \mathbf{C}_\alpha)$	stands for	$\mathrm{I}\, x : \alpha \;.$ $(\mathbf{A}_o \Rightarrow x = \mathbf{B}_\alpha) \wedge (\neg \mathbf{A}_o \Rightarrow x = \mathbf{C}_\alpha)$ where $\alpha \neq o$.
$(\mathbf{A}_o \mapsto \mathbf{B}_\alpha \mid \mathbf{C}_\alpha)$	stands for	$\mathsf{IF}(\mathbf{A}_o, \mathbf{B}_\alpha, \mathbf{C}_\alpha)$

Sections 6.1, 6.4, and 6.8 must be modified to accommodate this change. Also, on p. 203 (Subsection 14.2.3), the notational definitions for the pseudoconstant $\mathsf{if}_{o \to \sigma \to \tau \to \alpha}$ and the conditional expression $(\mathbf{A}_o \mapsto \mathbf{B}_\sigma \mid \mathbf{C}_\tau)$ should be removed from Table 14.3.

Corrigendum 5

On p. 68 (Section 6.2), the condition for the notational definition for

$$(\exists!\, \mathbf{x} : \alpha \;.\; \mathbf{A}_o)$$

given in Table 6.3 should be

where y does not occur in $(\lambda\, \mathbf{x} : \alpha \;.\; \mathbf{A}_o)$

instead of

where y is not free in $(\lambda\, \mathbf{x} : \alpha \;.\; \mathbf{A}_o)$.

Corrigendum 6

On p. 74 (Section 6.9), the notational definitions for $(\mathbf{Q}_{\{\alpha\}} \to \mathbf{R}_{\{\beta\}})$, $(\alpha \to \mathbf{R}_{\{\beta\}})$, and $(\mathbf{Q}_{\{\alpha\}} \to \beta)$ given in Table 6.9 require the condition

> where $\beta \neq o$.

To complete this correction, the following notational definitions need to be added to Table 6.9:

$(\mathbf{Q}_{\{\alpha\}} \to o)$	stands for	$\{s : \{\alpha\} \mid s \subseteq \mathbf{Q}_{\{\alpha\}}\}$.
$\mathcal{P}(\mathbf{Q}_{\{\alpha\}})$	stands for	$\mathbf{Q}_{\{\alpha\}} \to o$.

Corrigendum 7

The introductory remarks in Example 9.6 on p. 98 (Section 9.1) say

> ... Giuseppe Peano presented a characterization of the natural numbers based on 0 and the successor function S. We express his characterization in Alonzo as the following theory PA called *Peano Arithmetic*:

However, this is not true: Peano's characterization of the natural numbers starts with 1, not 0. Thus the introductory remarks should instead say

> ... Giuseppe Peano presented a characterization of the natural numbers based on 1 and the successor function S. We express his characterization in Alonzo as the following theory PA (in which we start with 0 instead of 1) called *Peano Arithmetic*:

Corrigendum 8

Example 4.47 on p. 114 and the proof of Theorem 9.49 on p. 115 (both in Section 9.4) assume, without proof, that there is a standard model of RAT. This hole in the example can be eliminated by simplifying RAT and then reorganizing the proof that RAT is categorical in the standard sense as follows:

Example 9.47 (Rational Numbers Order) In Example 9.18, we defined DWTOWE, a theory of dense weak total orders without endpoints. Let us now define an extension RAT of DWTOWE called *Rational Numbers Order*.

Theory Extension 9.48 (Rational Numbers Order)

Name: RAT

Extends DWTOWE

New base types: (none)

New constants: (none)

New axioms:

9. COUNT$(U_{\{S\}})$. (S is countable.)

All dense weak total orders without endpoints are infinite and either countable, such as $(\mathbb{Q}, \leq)$, or uncountable, such as $(\mathbb{R}, \leq)$. Let M be a standard model of DWTOWE. Then, by virtue of axiom 9, M is model of RAT iff D_S^M is countable. Hence the standard model M_{rat} of DWTOWE that defines $(\mathbb{Q}, \leq)$ is a model of RAT. □

Proposition 9.49 RAT *is a specification in the standard sense of all countable dense weak total orders without endpoints.*

Proof This follows from the fact given in Example 9.47 that a standard model M of DWTOWE is a model of RAT iff D_S^M is countable. □

Theorem 9.50 (Categoricity of RAT) RAT *is categorical in the standard sense.*

Proof As shown in Example 9.47, the standard model M_{rat} of DWTOWE that defines $(\mathbb{Q}, \leq)$ is a model of RAT. Georg Cantor proved in 1895 that every two countable dense weak total orders without endpoints are order isomorphic [20]. Thus, by Proposition 9.49, every model of RAT is isomorphic to M_{rat}, and so RAT is categorical in the standard sense. □

Corrigendum 9

On p. 115 (Section 9.5), a theory $T = (L, \Gamma)$ is defined to be *(semantically) complete* if either $T \vDash \mathbf{A}_o$ or $T \vDash \neg\mathbf{A}_o$ holds for all sentences $\mathbf{A}_o$ of L. Hence every unsatisfiable theory is incomplete. Therefore, Example 9.51 (Theory of False) and Proposition 9.52 are incorrect and should be removed.

Corrigendum 10

On p. 126 (Section 9.9), the hint for Exercise 9.9.17 is misleading and should be removed.

Corrigendum 11

On p. 131 (Section 11.1), the definition of a theorem package P being valid in a theory should be:

> P is *valid in a theory* $T = (L, \Gamma)$ if $\mathbf{A}_o$ is a sentence of L and π is a proof of $\mathbf{A}_o$ from Γ.

That is, $\mathbf{A}_o$ is required to be a sentence of L.

Corrigendum 12

On p. 164 (Subsection 13.3.2), the statement and proof of Lemma 13.9 tacitly assume that $\overline{\mu}(\alpha)$ is defined, but $\overline{\mu}(\alpha)$ may be undefined in some cases. The lemma and its proof are corrected as follows:

Lemma 13.9 *Let* $\Phi = (\mu, \nu)$ *be a translation from* T_1 *to* T_2.

1. *If* $\alpha \in \mathcal{T}(L_1)$ *and* $\overline{\mu}(\alpha) \in \mathcal{Q}_2$, *then* $T_2 \models \overline{\mu}(\alpha)\!\downarrow \Rightarrow \overline{\nu}(U_{\{\alpha\}}) = \overline{\mu}(\alpha)$.
2. *If* $\alpha \in \mathcal{T}(L_1)$ *and* $\overline{\mu}(\alpha) \in \mathcal{Q}_2$, *then* $T_2 \models \overline{\mu}(\alpha)\!\uparrow \Rightarrow \overline{\nu}(U_{\{\alpha\}}) = \emptyset_{\{\tau(\overline{\mu}(\alpha))\}}$.
3. *If* $\mathbf{a} \in \mathcal{B}_1$ *and* $\mu(\mathbf{a}) \in \mathcal{Q}_2$, *then*
$$T_2 \models \overline{\nu}(U_{\{\mathbf{a}\}} \neq \emptyset_{\{\mathbf{a}\}}) \Leftrightarrow (\mu(\mathbf{a})\!\downarrow \wedge\, \mu(\mathbf{a}) \neq \emptyset_{\{\tau(\mu(\mathbf{a}))\}}).$$
4. *If* $\mathbf{c}_\alpha \in \mathcal{C}_1$ *and* $\overline{\mu}(\alpha) \in \mathcal{T}_2$, *then* $T_2 \models \overline{\nu}(\mathbf{c}_\alpha \downarrow U_{\{\alpha\}}) \Leftrightarrow \nu(\mathbf{c}_\alpha)\!\downarrow$.
5. *If* $\mathbf{c}_\alpha \in \mathcal{C}_1$ *and* $\overline{\mu}(\alpha) \in \mathcal{Q}_2$, *then*
$$T_2 \models \overline{\mu}(\alpha)\!\downarrow \Rightarrow \overline{\nu}(\mathbf{c}_\alpha \downarrow U_{\{\alpha\}}) \Leftrightarrow \nu(\mathbf{c}_\alpha) \downarrow \overline{\mu}(\alpha).$$

Proof Let $\alpha \in \mathcal{T}(L_1)$, $\overline{\mu}(\alpha) \in \mathcal{Q}_2$, and ($\star$) M be a general model of T_2 in which $\overline{\mu}(\alpha)\!\downarrow$ is true. We must show that $\overline{\nu}(U_{\{\alpha\}}) = \overline{\mu}(\alpha)$ is true in M. Then

$$\begin{aligned}
& \overline{\nu}(U_{\{\alpha\}}) \\
\equiv\ & \overline{\nu}(\lambda\, x : \alpha \,.\, T_o) \\
\equiv\ & \lambda\, x : \overline{\mu}(\alpha) \,.\, T_o \\
\equiv\ & \lambda\, x : \tau(\overline{\mu}(\alpha)) \,.\, (x \in \overline{\mu}(\alpha) \mapsto T_o \mid F_o).
\end{aligned}$$

by the definition of $\overline{\nu}$ and notational definitions. The last expression is clearly equal to $\lambda\, x : \tau(\overline{\mu}(\alpha))\ .\ \overline{\mu}(\alpha)\, x$, which is equal to $\overline{\mu}(\alpha)$ in M by ($\star$). This proves part 1. Part 2 follows from the proof of part 1 and the notational definition for the empty set pseudoconstant. Part 3 follows immediately from the definition of $\overline{\nu}$ and parts 1 and 2. Part 4 follows from the definition of $\overline{\nu}$ and the notational definition for the defined-in-quasitype operator. And part 5 follows immediately from the definition of $\overline{\nu}$ and part 1. □

Corrigendum 13

On p. 166 (Subsection 13.3.2), the proof of Theorem 13.13 (Morphism Theorem) is incorrect for nonnormal translations. The extraction of the structure M_1 from a model M_2 of T_2 is more complicated when Φ is not a normal translation and it takes more work to prove that, if each obligation of Φ is true in M_2, then M_1 is a model of T_1.

More specifically, the text between Lemma 13.11 and Theorem 13.13 needs be replaced with the following text (which will require the subsequence theorems and examples to be renumbered):

Let $\Phi = (\mu, \nu)$ be a translation from T_1 to T_2,

$$M_2 = (\{D^2_\alpha \mid \alpha \in \mathcal{T}_2\}, I_2)$$

be a model of T_2 in which each obligation of Φ of the first and second kind is true, and $\varphi \in \mathsf{assign}(M_2)$. We will extract an interpretation M_1 from M_2 using Φ as follows. To start, let us define

$$D_\alpha = \begin{cases} D^2_{\overline{\mu}(\alpha)} & \text{if } \overline{\mu}(\alpha) \in \mathcal{T}_2 \\ \{d \in D^2_{\tau(\overline{\mu}(\alpha))} \mid d \in V^{M_2}_\varphi(\overline{\mu}(\alpha))\} & \text{if } \overline{\mu}(\alpha) \in \mathcal{Q}_2 \end{cases}$$

for $\alpha \in \mathcal{T}_1$.

Lemma 13.12 *D_α is nonempty for all $\alpha \in \mathcal{T}_1$.*

Proof The proof is by induction on the syntactic structure of types. D_o is obviously nonempty. If $\overline{\mu}(\mathbf{a}) \in \mathcal{T}_2$, then $D_{\mathbf{a}}$ is also obviously nonempty. If $\overline{\mu}(\mathbf{a}) \in \mathcal{Q}_2$, then $D_{\mathbf{a}}$ is nonempty by part 3 of Lemma 13.9 and the fact that the obligations of Φ of the first kind are true in M_2. If $\alpha = \beta \rightarrow \gamma$ and D_β and D_γ are nonempty, then D_α is nonempty since it contains the empty function. If $\alpha = \beta \times \gamma$ and D_β and D_γ are nonempty, then D_α is nonempty since it equals $D_\beta \times D_\gamma$. □

If Φ is normal, then $\{D_\alpha \mid \alpha \in \mathcal{T}_1\}$ is clearly a frame for L_1. On the other hand, if Φ is not normal, then $\{D_\alpha \mid \alpha \in \mathcal{T}_1\}$ may not be a frame. This will happen because $D_{\alpha\to\beta} \subseteq D^2_{\tau(\overline{\mu}(\alpha))} \to D^2_{\tau(\overline{\mu}(\beta))}$ and thus $D_{\alpha\to\beta} \not\subseteq D_\alpha \to D_\beta$ if $D_\alpha \subset D^2_{\tau(\overline{\mu}(\alpha))}$ or $D_\beta \subset D^2_{\tau(\overline{\mu}(\beta))}$. However, $\mathsf{dom}(f) \subseteq D_\alpha$ and $\mathsf{ran}(f) \subseteq D_\beta$ for all $f \in D_{\alpha\to\beta}$. Therefore, we can turn a recalcitrant $\{D_\alpha \mid \alpha \in \mathcal{T}_1\}$ of this kind into a frame by modifying the members of its function domains as follows.

We will define D^1_α and $H_\alpha : D_\alpha \to D^1_\alpha$ for all $\alpha \in \mathcal{T}_1$ by recursion on the syntactic structure of types. We will also prove that H_α is a bijection for all $\alpha \in \mathcal{T}_1$ by induction on the syntactic structure of types. There are four cases to consider:

Case 1: $\alpha = o$. Define $D^1_\alpha = D_\alpha$ and H_α to be the identity function. H_α is clearly a bijection.

Case 2: $\alpha = \mathbf{a}$. Define $D^1_\alpha = D_\alpha$ and H to be the identity function. H_α is clearly a bijection.

Case 3: $\alpha = \beta \to \gamma$. Assume D^1_β and D^1_γ are defined and $H_\beta : D_\beta \to D^1_\beta$ and $H_\gamma : D_\gamma \to D^1_\gamma$ are bijections. Define $H_\alpha(f)(x) \simeq H_\gamma(f(H^{-1}_\beta(x)))$[1] for all $f \in D_\alpha$ and $x \in D^1_\beta$ and $D^1_\alpha = H_\alpha[D_\alpha]$, i.e., the image of D_α under H_α. H_α is clearly a bijection since H_β and H_γ are bijections.

Case 4: $\alpha = \beta \times \gamma$. Assume D^1_β and D^1_γ are defined and $H_\beta : D_\beta \to D^1_\beta$ and $H_\gamma : D_\gamma \to D^1_\gamma$ are bijections. Define $D^1_\alpha = D^1_\beta \times D^1_\gamma$ and $H_\alpha((a,b)) = (H_\beta(a), H_\gamma(b))$. H_α is clearly a bijection since H_β and H_γ are bijections.

Lemma 13.13 *$\{D^1_\alpha \mid \alpha \in \mathcal{T}_1\}$ is a frame of L_1.*

Proof Follows from Lemma 13.12 and the construction of the D^1_α. □

Finally, define $I_1(\mathbf{c}_\alpha) \simeq H_\alpha(V^{M_2}_\varphi(\nu(\mathbf{c}_\alpha)))$[2] for $\mathbf{c}_\alpha \in \mathcal{C}_1$ and

$$M_1 = (\{D^1_\alpha \mid \alpha \in \mathcal{T}_1\}, I_1).$$

Lemma 13.14 *M_1 is an interpretation of L_1.*

[1] That is, $H_\alpha(f)(x) = H_\gamma(f(H^{-1}_\beta(x)))$ if $f(H^{-1}_\beta(x))$ is defined and $H_\alpha(f)(x)$ is undefined otherwise.

[2] That is, $I_1(\mathbf{c}_\alpha) = H_\alpha(V^{M_2}_\varphi(\nu(\mathbf{c}_\alpha)))$ if $V^{M_2}_\varphi(\nu(\mathbf{c}_\alpha))$ is defined and $I_1(\mathbf{c}_\alpha)$ is undefined otherwise.

Proof $\{D^1_\alpha \mid \alpha \in \mathcal{T}_1\}$ is a frame of L_1 by Lemma 13.13. Let $\mathbf{c}_\alpha \in \mathcal{C}_1$. If $\overline{\mu}(\alpha) \in \mathcal{T}_2$, then $I_1(\mathbf{c}_\alpha) = H_\alpha(V^{M_2}_\varphi(\nu(\mathbf{c}_\alpha))) = V^{M_2}_\varphi(\nu(\mathbf{c}_\alpha)) \in D^2_{\overline{\mu}(\alpha)} = D^1_\alpha$ by part 4 of Lemma 13.9 and the fact that the obligations of Φ of the second kind are true in M_2. If $\overline{\mu}(\alpha) \in \mathcal{Q}_2$, then $V^{M_2}_\varphi(\nu(\mathbf{c}_\alpha)) \in D^2_{\overline{\mu}(\alpha)}$ by part 5 of Lemma 13.9 and the fact that the obligations of Φ of the second kind are true in M_2, and thus $I_1(\mathbf{c}_\alpha) \in D^1_\alpha$ since H_α is a bijection. Therefore, M_1 is an interpretation of L_1. □

Lemma 13.15 *Let $\Phi = (\mu, \nu)$ be a translation from T_1 to T_2, M_2 be a model of T_2 in which each obligation of Φ is true in M_2, and M_1 be the structure defined above. Then M_1 is a model of T_1.*

Proof M_1 is an interpretation of L_1 by Lemma 13.14.

For all $\mathbf{A}_\alpha \in \mathcal{E}_1$ and $\varphi \in \mathsf{assign}(M_1)$, define

$(\star)$ $V^{M_1}_\varphi(\mathbf{A}_\alpha) \simeq H_\alpha(V^{M_2}_{\overline{\nu}(\varphi)}(\overline{\nu}(\mathbf{A}_\alpha)))$[3],

where $\overline{\nu}(\varphi)$ is any $\psi \in \mathsf{assign}(M_2)$ such that $\psi(\overline{\nu}((\mathbf{x} : \beta))) = H^{-1}_\beta(\varphi((\mathbf{x} : \beta)))$ for all variables $(\mathbf{x} : \beta) \in \mathcal{E}_1$. By induction on the syntactic structure of expressions, $V^{M_1}_\varphi$ satisfies the seven conditions of the definition of a general model. Therefore, M_1 is a general model of L_1.

$(\star)$ implies

$(\star\star)$ $M_1 \vDash \mathbf{A}_o$ iff $M_2 \vDash \overline{\nu}(\mathbf{A}_o)$ for all sentences $\mathbf{A}_o \in \mathcal{E}_1$.

If $\mathbf{A}_o \in \Gamma_1$, then $M_2 \vDash \overline{\nu}(\mathbf{A}_o)$ since each obligation of Φ of the third kind is true in M_2. Hence $M_1 \vDash \mathbf{A}_o$ for all $\mathbf{A}_o \in \Gamma_1$ by $(\star\star)$. Therefore, M_1 is a model of T_1. □

Corrigendum 14

The definition transportation and theorem transportation modules defined on p. 184 (Subsection 13.4.2) should include *Source development* and *Target development* fields.

Corrigendum 15

The F2 component of the definition of a frame for AlonzoS on p. 202 (Subsection 14.2.3) is not correctly formulated. It should say that $f(d) = \textsc{f}$ for all $f \in D_{\sigma \to o}$ and $d \in D_{\overline{\xi}(\sigma)} \setminus D_\sigma$. The following is the correct formulation:

[3]That is, $V^{M_1}_\varphi(\mathbf{A}_\alpha) = H_\alpha(V^{M_2}_{\overline{\nu}(\varphi)}(\overline{\nu}(\mathbf{A}_\alpha)))$ if $V^{M_2}_{\overline{\nu}(\varphi)}(\overline{\nu}(\mathbf{A}_\alpha))$ is defined and $V^{M_1}_\varphi(\mathbf{A}_\alpha)$ is undefined otherwise.

F2. *Predicate domain*: $D_{\sigma \rightarrow o}$ is a set of *some* total functions f from $D_{\bar{\xi}(\sigma)}$ to D_o such that $f(d) = \text{F}$ for all $d \in D_{\bar{\xi}(\sigma)} \setminus D_\sigma$ for $\sigma \in \mathcal{U}(L)$.

Corrigendum 16

The V9 component of the definition of a general model of AlonzoQE on p. 208 (Subsection 14.3.3) is not correct when $V_\varphi^M(\mathbf{A}_\epsilon)$ denotes an expression $\mathbf{E}_\alpha$ such that $V_\varphi^M(\mathbf{E}_\alpha)$ is undefined. The following is the correct formulation:

V9. $V_\varphi^M(\llbracket \mathbf{A}_\epsilon \rrbracket_\alpha) = V_\varphi^M(V_\varphi^M(\mathbf{A}_\epsilon))$ if $V_\varphi^M(\mathbf{A}_\epsilon)$ is an expression $\mathbf{E}_\alpha$ such that $V_\varphi^M(\mathbf{E}_\alpha)$ is defined. Otherwise, $V_\varphi^M(\llbracket \mathbf{A}_\epsilon \rrbracket_\alpha) = \text{F}$ if $\alpha = o$ and is undefined if $\alpha \neq o$.

To complete this correction, Theorem 14.9 and its proof must be changed as follows where several $=$ symbols are replaced with $\simeq$ symbols:

Theorem 14.9 (Law of Disquotation) *Let AlonzoQE be the logic. Then* $\llbracket \ulcorner \mathbf{A}_\alpha \urcorner \rrbracket_\alpha \simeq \mathbf{A}_\alpha$ *is valid.*

Proof Let $\mathbf{A}_\alpha$ be an eval-free expression of a language L, M be a general model of L, and $\varphi \in \mathsf{assign}(M)$. Then

$$
\begin{aligned}
& V_\varphi^M(\llbracket \ulcorner \mathbf{A}_\alpha \urcorner \rrbracket_\alpha) \\
\simeq\ & V_\varphi^M(V_\varphi^M(\ulcorner \mathbf{A}_\alpha \urcorner)) && (1) \\
\simeq\ & V_\varphi^M(\mathbf{A}_\alpha). && (2)
\end{aligned}
$$

(a) $V_\varphi^M(\ulcorner \mathbf{A}_\alpha \urcorner) = \mathbf{A}_\alpha$ by condition V8 of the definition of a general model. (a) implies (b) $V_\varphi^M(\ulcorner \mathbf{A}_\alpha \urcorner)$ is an expression of type α. (1) is by (b) and condition V9 of the definition of a general model; and (2) is by (a). Hence $V_\varphi^M(\llbracket \ulcorner \mathbf{A}_\alpha \urcorner \rrbracket_\alpha) \simeq V_\varphi^M(\mathbf{A}_\alpha)$ for all general models M that interpret $\mathbf{A}_\alpha$ and all $\varphi \in \mathsf{assign}(M)$, and so $\llbracket \ulcorner \mathbf{A}_\alpha \urcorner \rrbracket_\alpha \simeq \mathbf{A}_\alpha$ is valid by part 5 of Lemma 6.5. □

References

[1] C. Benzmüller and P. Andrews. Church's type theory. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, Summer 2019 edition, 2019. Available at http://plato.stanford.edu/archives/sum2019/entries/type-theory-church/.

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