LaTeX for Alonzo

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Abstract

This document illustrates how to use a set of LaTeX macros and environments for presenting types, expressions, and modules of Alonzo, a practice-oriented version of simple type theory.

Contents

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1 Introduction

Alonzo [1] is a practice-oriented logic based on Alonzo Church's formulation of *simple type theory* known as *Church's type theory.* The LaTeX source file

`alonzo-notation.tex`,

whose contents are given in Section 4, contains a set of LaTeX macros for presenting Alonzo types and expressions in both formal and compact notations. It also contains a set of LaTeX environments for presenting Alonzo mathematical knowledge modules. This document illustrates how to use these macros and environments with a series of examples.

2 Macros

In this section, each LaTeX macro in

`alonzo-notation.tex`

is presented with metavariable arguments followed by the output it produces.

2.1 Macros for Presenting Types in Formal Notation

1. `\fBoolTy`

   BoolTy

2. `\fBaseTy {\mathbf{a}}`

   $\mathsf{BaseTy}(\mathbf{a})$

3. `\fFunTy {\alpha} {\beta}`

   $\mathsf{FunTy}(\alpha, \beta)$

4. `\fProdTy {\alpha} {\beta}`

   $\mathsf{ProdTy}(\alpha, \beta)$

2.2 Macros for Presenting Expressions in Formal Notation

1. `\fVar {\mathbf{x}} {\alpha}`

   $\mathsf{Var}(\mathbf{x}, \alpha)$

2. `\fCon {\mathbf{c}} {\alpha}`

   $\mathsf{Con}(\mathbf{c}, \alpha)$

3. `\fEq {\mathbf{A}_\alpha} {\mathbf{B}_\alpha}`

   $\mathsf{Eq}(\mathbf{A}_\alpha, \mathbf{B}_\alpha)$

4. ```
   \fFunApp
     {\mathbf{F}_{\cFunTyX {\alpha} {\beta}}}
     {\mathbf{A}_\alpha}
   ```

   $\mathsf{FunApp}(\mathbf{F}_{\alpha \rightarrow \beta}, \mathbf{A}_\alpha)$

5. `\fFunAbs {\mathbf{x}} {\alpha} {\mathbf{B}_\beta}`

   $\mathsf{FunAbs}(\mathsf{Var}(\mathbf{x}, \alpha), \mathbf{B}_\beta)$

6. `\fDefDes {\mathbf{x}} {\alpha} {\mathbf{A}_{\cB}}`

   $\mathsf{DefDes}(\mathsf{Var}(\mathbf{x}, \alpha), \mathbf{A}_o)$

7. `\fOrdPair {\mathbf{A}_\alpha} {\mathbf{B}_\beta}`

   $\mathsf{OrdPair}(\mathbf{A}_\alpha, \mathbf{B}_\beta)$
   ```

2.3 Macros for Presenting Types in Compact Notation

1. `\cBoolTy` or `\cB`

   o

2. `\cBaseTy {\mathbf{a}}`

   $\mathbf{a}$

3. `\cFunTy {\alpha} {\beta}`

   $(\alpha \to \beta)$

4. `\cFunTyX {\alpha} {\beta}`

   $\alpha \to \beta$

5. `\cFunTyB {\alpha} {\beta} {\gamma}`

   $(\alpha \to \beta \to \gamma)$

6. `\cFunTyBX {\alpha} {\beta} {\gamma}`

   $\alpha \to \beta \to \gamma$

7. `\cFunTyC {\alpha} {\beta} {\gamma} {\delta}`

   $(\alpha \to \beta \to \gamma \to \delta)$

8. `\cFunTyCX {\alpha} {\beta} {\gamma} {\delta}`

   $\alpha \to \beta \to \gamma \to \delta$

9. `\cProdTy {\alpha} {\beta}`

   $(\alpha \times \beta)$

10. `\cProdTyX {\alpha} {\beta}`

    $\alpha \times \beta$

11. `\cProdTyB {\alpha} {\beta} {\gamma}`

    $(\alpha \times \beta \times \gamma)$

12. `\cProdTyBX {\alpha} {\beta} {\gamma}`

    $\alpha \times \beta \times \gamma$

13. `\cProdTyC {\alpha} {\beta} {\gamma} {\delta}`

    $(\alpha \times \beta \times \gamma \times \delta)$

14. `\cProdTyCX {\alpha} {\beta} {\gamma} {\delta}`

    $\alpha \times \beta \times \gamma \times \delta$

2.4 Macros for Presenting Expressions in Compact Notation

1. `\cVar {\mathbf{x}} {\alpha}`

   $(\mathbf{x} : \alpha)$

2. `\cVarY {\mathbf{x}} {\alpha}`

   $\mathbf{x}$

3. `\cCon {\mathbf{c}} {\alpha}`

   $\mathbf{c}_\alpha$

4. `\cConY {\mathbf{c}} {\alpha}`

   $\mathbf{c}$

5. `\cEq {\mathbf{A}_\alpha} {\mathbf{B}_\alpha}`

   $(\mathbf{A}_\alpha = \mathbf{B}_\alpha)$

6. `\cEqX {\mathbf{A}_\alpha} {\mathbf{B}_\alpha}`

   $\mathbf{A}_\alpha = \mathbf{B}_\alpha$

7. ```
   \cFunApp
     {\mathbf{F}_{\cFunTyX {\alpha} {\beta}}}
     {\mathbf{A}_\alpha}
   ```

   $(\mathbf{F}_{\alpha \rightarrow \beta}\, \mathbf{A}_\alpha)$

8. ```
   \cFunAppX
     {\mathbf{F}_{\cFunTyX {\alpha} {\beta}}}
     {\mathbf{A}_\alpha}
   ```

   $\mathbf{F}_{\alpha \rightarrow \beta}\, \mathbf{A}_\alpha$

9. ```
   \cFunAppB
     {\mathbf{F}_{\cFunTyBX {\alpha} {\beta} {\gamma}}}
     {\mathbf{A}_\alpha}
     {\mathbf{B}_\beta}
   ```

   $(\mathbf{F}_{\alpha \rightarrow \beta \rightarrow \gamma}\, \mathbf{A}_\alpha\, \mathbf{B}_\beta)$
   ```

10. `\cFunAppBX`
```
{\mathbf{F}_{\cFunTyBX {\alpha} {\beta} {\gamma}}}
{\mathbf{A}_\alpha}
{\mathbf{B}_\beta}
```
$\mathbf{F}_{\alpha \to \beta \to \gamma} \, \mathbf{A}_\alpha \, \mathbf{B}_\beta$

11. `\cFunAppC`
```
{\mathbf{F}_{\cFunTyCX
  {\alpha}
  {\beta}
  {\gamma}
  {\delta}}}
{\mathbf{A}_\alpha}
{\mathbf{B}_\beta}
{\mathbf{C}_\gamma}
```
$(\mathbf{F}_{\alpha \to \beta \to \gamma \to \delta} \, \mathbf{A}_\alpha \, \mathbf{B}_\beta \, \mathbf{C}_\gamma)$

12. `\cFunAppCX`
```
{\mathbf{F}_{\cFunTyCX
  {\alpha}
  {\beta}
  {\gamma}
  {\delta}}}
{\mathbf{A}_\alpha}
{\mathbf{B}_\beta}
{\mathbf{C}_\gamma}
```
$\mathbf{F}_{\alpha \to \beta \to \gamma \to \delta} \, \mathbf{A}_\alpha \, \mathbf{B}_\beta \, \mathbf{C}_\gamma$

13. `\cFunAbs {\mathbf{x}} {\alpha} {\mathbf{B}_\beta}`

    $(\lambda \, \mathbf{x} : \alpha \, . \, \mathbf{B}_\beta)$

14. `\cFunAbsX {\mathbf{x}} {\alpha} {\mathbf{B}_\beta}`

    $\lambda \, \mathbf{x} : \alpha \, . \, \mathbf{B}_\beta$

15. `\cDefDes {\mathbf{x}} {\alpha} {\mathbf{A}_{\cB}}`

    $(\mathrm{I} \, \mathbf{x} : \alpha \, . \, \mathbf{A}_o)$

16. `\cDefDesX {\mathbf{x}} {\alpha} {\mathbf{A}_{\cB}}`

    $\mathrm{I} \, \mathbf{x} : \alpha \, . \, \mathbf{A}_o$

17. `\cOrdPair {\mathbf{A}_\alpha} {\mathbf{B}_\beta}`

    $(\mathbf{A}_\alpha, \mathbf{B}_\beta)$

2.5 Macros for Boolean Operators

1. `\cTPC` or `\cT`

   T_o

2. `\cFPC` or `\cF`

   F_o

3. `\cAndPC`

   $\wedge_{o \to o \to o}$

4. `\cAnd {\mathbf{A}_\cB} {\mathbf{B}_\cB}`

   $(\mathbf{A}_o \wedge \mathbf{B}_o)$

5. `\cAndX {\mathbf{A}_\cB} {\mathbf{B}_\cB}`

   $\mathbf{A}_o \wedge \mathbf{B}_o$

6. ```
   \cAndB
     {\mathbf{A}_\cB}
     {\mathbf{B}_\cB}
     {\mathbf{C}_\cB}
   ```

   $(\mathbf{A}_o \wedge \mathbf{B}_o \wedge \mathbf{C}_o)$

7. ```
   \cAndBX
     {\mathbf{A}_\cB}
     {\mathbf{B}_\cB}
     {\mathbf{C}_\cB}
   ```

   $\mathbf{A}_o \wedge \mathbf{B}_o \wedge \mathbf{C}_o$

8. ```
   \cAndL
     {\mathbf{A}^{1}_{\cB}
      \And
      \cdots
      \And
      \mathbf{A}^{n}_{\cB}}
   ```

   $(\mathbf{A}^1_o \;\&\; \cdots \;\&\; \mathbf{A}^n_o)$
   ```

9. ```
\cAndLX
  {\mathbf{A}^{1}_{\cB}
   \And
   \cdots
   \And
   \mathbf{A}^{n}_{\cB}}
```

   $\mathbf{A}^1_o \;\&\; \cdots \;\&\; \mathbf{A}^n_o$

10. `\cImpliesPC`

    $\Rightarrow_{o\to o\to o}$

11. `\cImplies {\mathbf{A}_\cB} {\mathbf{B}_\cB}`

    $(\mathbf{A}_o \Rightarrow \mathbf{B}_o)$

12. `\cImpliesX {\mathbf{A}_\cB} {\mathbf{B}_\cB}`

    $\mathbf{A}_o \Rightarrow \mathbf{B}_o$

13. `\cNegPC`

    $\neg_{o\to o}$

14. `\cNeg {\mathbf{A}_\cB}`

    $(\neg \mathbf{A}_o)$

15. `\cNegX {\mathbf{A}_\cB}`

    $\neg \mathbf{A}_o$

16. `\cOrPC`

    $\vee_{o\to o\to o}$

17. `\cOr {\mathbf{A}_\cB} {\mathbf{B}_\cB}`

    $(\mathbf{A}_o \vee \mathbf{B}_o)$

18. `\cOrX {\mathbf{A}_\cB} {\mathbf{B}_\cB}`

    $\mathbf{A}_o \vee \mathbf{B}_o$

19. ```
\cOrB
  {\mathbf{A}_\cB}
  {\mathbf{B}_\cB}
  {\mathbf{C}_\cB}
```

    $(\mathbf{A}_o \vee \mathbf{B}_o \vee \mathbf{C}_o)$

20. \cOrBX
```
{\mathbf{A}_\cB}
{\mathbf{B}_\cB}
{\mathbf{C}_\cB}
```
$\mathbf{A}_o \vee \mathbf{B}_o \vee \mathbf{C}_o$

21. \cOrL
```
{\mathbf{A}^{1}_{\cB}
 \Or
 \cdots
 \Or
 \mathbf{A}^{n}_{\cB}}
```
$(\mathbf{A}^1_o \vee \cdots \vee \mathbf{A}^n_o)$

22. \cOrLX
```
{\mathbf{A}^{1}_{\cB}
 \Or
 \cdots
 \Or
 \mathbf{A}^{n}_{\cB}}
```
$\mathbf{A}^1_o \vee \cdots \vee \mathbf{A}^n_o$

2.6 Macros for Binary Operators

1. \cBin
```
{\mathbf{A}_\alpha}
{\cCon
   {\mathbf{c}}
   {\cFunTyBX {\alpha} {\alpha} {\beta}}}
{\mathbf{B}_\alpha}
```
$(\mathbf{A}_\alpha\ \mathbf{c}_{\alpha\to\alpha\to\beta}\ \mathbf{B}_\alpha)$

2. \cBinX
```
{\mathbf{A}_\alpha}
{\cCon
   {\mathbf{c}}
   {\cFunTyBX {\alpha} {\alpha} {\beta}}}
{\mathbf{B}_\alpha}
```
$\mathbf{A}_\alpha\ \mathbf{c}_{\alpha\to\alpha\to\beta}\ \mathbf{B}_\alpha$

3. ```
\cBinB
  {\mathbf{A}_\alpha}
  {\cCon
     {\mathbf{c}}
     {\cFunTyBX {\alpha} {\alpha} {\beta}}}
  {\mathbf{B}_\alpha}
  {\cCon
     {\mathbf{d}}
     {\cFunTyBX {\alpha} {\alpha} {\beta}}}
  {\mathbf{C}_\alpha}
```
   $(\mathbf{A}_\alpha\ \mathbf{c}_{\alpha\to\alpha\to\beta}\ \mathbf{B}_\alpha\ \mathbf{d}_{\alpha\to\alpha\to\beta}\ \mathbf{C}_\alpha)$

4. ```
\cBinBX
  {\mathbf{A}_\alpha}
  {\cCon
     {\mathbf{c}}
     {\cFunTyBX {\alpha} {\alpha} {\beta}}}
  {\mathbf{B}_\alpha}
  {\cCon
     {\mathbf{d}}
     {\cFunTyBX {\alpha} {\alpha} {\beta}}}
  {\mathbf{C}_\alpha}
```
   $\mathbf{A}_\alpha\ \mathbf{c}_{\alpha\to\alpha\to\beta}\ \mathbf{B}_\alpha\ \mathbf{d}_{\alpha\to\alpha\to\beta}\ \mathbf{C}_\alpha$

5. `\cIff {\mathbf{A}_\cB} {\mathbf{B}_\cB}`

   $(\mathbf{A}_o \Leftrightarrow \mathbf{B}_o)$

6. `\cIffX {\mathbf{A}_\cB} {\mathbf{B}_\cB}`

   $\mathbf{A}_o \Leftrightarrow \mathbf{B}_o$

7. `\cNotEq {\mathbf{A}_\alpha} {\mathbf{B}_\alpha}`

   $(\mathbf{A}_\alpha \neq \mathbf{B}_\alpha)$

8. `\cNotEqX {\mathbf{A}_\alpha} {\mathbf{B}_\alpha}`

   $\mathbf{A}_\alpha \neq \mathbf{B}_\alpha$

2.7 Macros for Quantifiers

1. `\cForall {\mathbf{x}} {\alpha} {\mathbf{A}_\cB}`

   $(\forall\, \mathbf{x} : \alpha\ .\ \mathbf{A}_o)$

2. ```
   \cForallX {\mathbf{x}} {\alpha} {\mathbf{A}_\cB}
   ```
   $\forall\, \mathbf{x} : \alpha \,.\, \mathbf{A}_o$

3. ```
   \cForallB
     {\mathbf{x}}
     {\alpha}
     {\mathbf{y}}
     {\beta}
     {\mathbf{A}_\cB}
   ```
   $(\forall\, \mathbf{x} : \alpha,\, \mathbf{y} : \beta \,.\, \mathbf{A}_o)$

4. ```
   \cForallBX
     {\mathbf{x}}
     {\alpha}
     {\mathbf{y}}
     {\beta}
     {\mathbf{A}_\cB}
   ```
   $\forall\, \mathbf{x} : \alpha,\, \mathbf{y} : \beta \,.\, \mathbf{A}_o$

5. ```
   \cForallC
     {\mathbf{x}}
     {\alpha}
     {\mathbf{y}}
     {\beta}
     {\mathbf{z}}
     {\gamma}
     {\mathbf{A}_\cB}
   ```
   $(\forall\, \mathbf{x} : \alpha,\, \mathbf{y} : \beta,\, \mathbf{z} : \gamma \,.\, \mathbf{A}_o)$

6. ```
   \cForallCX
     {\mathbf{x}}
     {\alpha}
     {\mathbf{y}}
     {\beta}
     {\mathbf{z}}
     {\gamma}
     {\mathbf{A}_\cB}
   ```
   $\forall\, \mathbf{x} : \alpha,\, \mathbf{y} : \beta,\, \mathbf{z} : \gamma \,.\, \mathbf{A}_o$
   ```

7. `\cForsome {\mathbf{x}} {\alpha} {\mathbf{A}_\cB}`

   $(\exists\, \mathbf{x} : \alpha\ .\ \mathbf{A}_o)$

8. `\cForsomeX {\mathbf{x}} {\alpha} {\mathbf{A}_\cB}`

   $\exists\, \mathbf{x} : \alpha\ .\ \mathbf{A}_o$

9. ```
   \cForsomeB
     {\mathbf{x}}
     {\alpha}
     {\mathbf{y}}
     {\beta}
     {\mathbf{A}_\cB}
   ```

   $(\exists\, \mathbf{x} : \alpha,\, \mathbf{y} : \beta\ .\ \mathbf{A}_o)$

10. ```
    \cForsomeBX
      {\mathbf{x}}
      {\alpha}
      {\mathbf{y}}
      {\beta}
      {\mathbf{A}_\cB}
    ```

    $\exists\, \mathbf{x} : \alpha,\, \mathbf{y} : \beta\ .\ \mathbf{A}_o$

11. ```
    \cForsomeC
      {\mathbf{x}}
      {\alpha}
      {\mathbf{y}}
      {\beta}
      {\mathbf{z}}
      {\gamma}
      {\mathbf{A}_\cB}
    ```

    $(\exists\, \mathbf{x} : \alpha,\, \mathbf{y} : \beta,\, \mathbf{z} : \gamma\ .\ \mathbf{A}_o)$

12. ```
    \cForsomeCX
      {\mathbf{x}}
      {\alpha}
      {\mathbf{y}}
      {\beta}
      {\mathbf{z}}
      {\gamma}
    ```

```
{\mathbf{A}_\cB}
```

$\exists\, \mathbf{x} : \alpha, \mathbf{y} : \beta, \mathbf{z} : \gamma \mathrel{.} \mathbf{A}_o$

13. `\cForsomeUnique`
    ```
    {\mathbf{x}}
    {\alpha}
    {\mathbf{A}_\cB}
    ```
    $(\exists!\, \mathbf{x} : \alpha \mathrel{.} \mathbf{A}_o)$

14. `\cForsomeUniqueX`
    ```
    {\mathbf{x}}
    {\alpha}
    {\mathbf{A}_\cB}
    ```
    $\exists!\, \mathbf{x} : \alpha \mathrel{.} \mathbf{A}_o$

2.8 Macros for Definedness

1. `\cBotPC {\alpha}`

   $\bot_\alpha$

2. `\cEmpFunPC {\alpha} {\beta}`

   $\Delta_{\alpha \rightarrow \beta}$

3. `\cIsDef {\mathbf{A}_\alpha}`

   $(\mathbf{A}_\alpha\!\downarrow)$

4. `\cIsDefX {\mathbf{A}_\alpha}`

   $\mathbf{A}_\alpha\!\downarrow$

5. `\cIsUndef {\mathbf{A}_\alpha}`

   $(\mathbf{A}_\alpha\!\uparrow)$

6. `\cIsUndefX {\mathbf{A}_\alpha}`

   $\mathbf{A}_\alpha\!\uparrow$

7. `\cQuasiEq {\mathbf{A}_\alpha} {\mathbf{B}_\alpha}`

   $(\mathbf{A}_\alpha \simeq \mathbf{B}_\alpha)$

8. `\cQuasiEqX {\mathbf{A}_\alpha} {\mathbf{B}_\alpha}`

   $\mathbf{A}_\alpha \simeq \mathbf{B}_\alpha$

9. `\cNotQuasiEq {\mathbf{A}_\alpha} {\mathbf{B}_\alpha}`

   $(\mathbf{A}_\alpha \not\simeq \mathbf{B}_\alpha)$

10. `\cNotQuasiEqX {\mathbf{A}_\alpha} {\mathbf{B}_\alpha}`

    $\mathbf{A}_\alpha \not\simeq \mathbf{B}_\alpha$

11. ```
    \cIfThenElse
      {\mathbf{A}_\cB}
      {\mathbf{B}_\alpha}
      {\mathbf{C}_\alpha}
    ```

    $\mathsf{IF}(\mathbf{A}_o, \mathbf{B}_\alpha, \mathbf{C}_\alpha)$

12. ```
    \cIf
      {\mathbf{A}_\cB}
      {\mathbf{B}_\alpha}
      {\mathbf{C}_\alpha}
    ```

    $(\mathbf{A}_o \mapsto \mathbf{B}_\alpha \mid \mathbf{C}_\alpha)$

13. ```
    \cIfX
      {\mathbf{A}_\cB}
      {\mathbf{B}_\alpha}
      {\mathbf{C}_\alpha}
    ```

    $\mathbf{A}_o \mapsto \mathbf{B}_\alpha \mid \mathbf{C}_\alpha$

2.9 Macros for Sets

1. `\cSetTy {\alpha}`

   $\{\alpha\}$

2. ```
   \cIn
     {\mathbf{A}_\alpha}
     {\mathbf{B}_{\cSetTy {\alpha}}}
   ```

   $(\mathbf{A}_\alpha \in \mathbf{B}_{\{\alpha\}})$

3. ```
   \cInX
     {\mathbf{A}_\alpha}
     {\mathbf{B}_{\cSetTy {\alpha}}}
   ```

   $\mathbf{A}_\alpha \in \mathbf{B}_{\{\alpha\}}$
   ```

4. `\cNotIn`
   ```
   {\mathbf{A}_\alpha}
   {\mathbf{B}_{\cSetTy {\alpha}}}
   ```
   $(\mathbf{A}_\alpha \notin \mathbf{B}_{\{\alpha\}})$

5. `\cNotInX`
   ```
   {\mathbf{A}_\alpha}
   {\mathbf{B}_{\cSetTy {\alpha}}}
   ```
   $\mathbf{A}_\alpha \notin \mathbf{B}_{\{\alpha\}}$

6. `\cSet {\mathbf{x}} {\alpha} {\mathbf{A}_\alpha}`

   $\{\mathbf{x} : \alpha \mid \mathbf{A}_\alpha\}$

7. `\cEmpSetPC {\alpha}`

   $\emptyset_{\{\alpha\}}$

8. `\cEmpSetAltPC {\alpha}`

   $\{\,\}_{\{\alpha\}}$

9. `\cUnivSetPC {\alpha}`

   $U_{\{\alpha\}}$

10. `\cFinSet {n} {\alpha}`

    n-α-SET

11. `\cFinSetL`
    ```
    {\mathbf{A}^{1}_{\alpha}
     ,
     \ldots
     ,
     \mathbf{A}^{n}_{\alpha}}
    ```
    $\{\mathbf{A}^1_\alpha, \ldots, \mathbf{A}^n_\alpha\}$

12. `\cSubseteqPC {\alpha}`

    $\subseteq_{\{\alpha\}\to\{\alpha\}\to o}$

13. `\cSubseteq`
    ```
    {\mathbf{A}_{\cSetTy {\alpha}}}
    {\mathbf{B}_{\cSetTy {\alpha}}}
    ```
    $(\mathbf{A}_{\{\alpha\}} \subseteq \mathbf{B}_{\{\alpha\}})$

14. ```
\cSubseteqX
  {\mathbf{A}_{\cSetTy {\alpha}}}
  {\mathbf{B}_{\cSetTy {\alpha}}}
```

    $\mathbf{A}_{\{\alpha\}} \subseteq \mathbf{B}_{\{\alpha\}}$

15. `\cUnionPC {\alpha}`

    $\cup_{\{\alpha\}\to\{\alpha\}\to\{\alpha\}}$

16. ```
\cUnion
  {\mathbf{A}_{\cSetTy {\alpha}}}
  {\mathbf{B}_{\cSetTy {\alpha}}}
```

    $(\mathbf{A}_{\{\alpha\}} \cup \mathbf{B}_{\{\alpha\}})$

17. ```
\cUnionX
  {\mathbf{A}_{\cSetTy {\alpha}}}
  {\mathbf{B}_{\cSetTy {\alpha}}}
```

    $\mathbf{A}_{\{\alpha\}} \cup \mathbf{B}_{\{\alpha\}}$

18. `\cIntersPC {\alpha}`

    $\cap_{\{\alpha\}\to\{\alpha\}\to\{\alpha\}}$

19. ```
\cInters
  {\mathbf{A}_{\cSetTy {\alpha}}}
  {\mathbf{B}_{\cSetTy {\alpha}}}
```

    $(\mathbf{A}_{\{\alpha\}} \cap \mathbf{B}_{\{\alpha\}})$

20. ```
\cIntersX
  {\mathbf{A}_{\cSetTy {\alpha}}}
  {\mathbf{B}_{\cSetTy {\alpha}}}
```

    $\mathbf{A}_{\{\alpha\}} \cap \mathbf{B}_{\{\alpha\}}$

21. `\cComplPC {\alpha}`

    $\overline{\cdot}_{\{\alpha\}\to\{\alpha\}}$

22. `\cCompl {\mathbf{A}_{\cSetTy {\alpha}}}`

    $\left(\overline{\mathbf{A}_{\{\alpha\}}}\right)$

23. `\cComplX {\mathbf{A}_{\cSetTy {\alpha}}}`

    $\overline{\mathbf{A}_{\{\alpha\}}}$
```

24. `\cSetDiffPC {\alpha}`

    $\setminus_{\{\alpha\}\to\{\alpha\}\to\{\alpha\}}$

25.
```
\cSetDiff
  {\mathbf{A}_{\cSetTy {\alpha}}}
  {\mathbf{B}_{\cSetTy {\alpha}}}
```

    $(\mathbf{A}_{\{\alpha\}} \setminus \mathbf{B}_{\{\alpha\}})$

26.
```
\cSetDiffX
  {\mathbf{A}_{\cSetTy {\alpha}}}
  {\mathbf{B}_{\cSetTy {\alpha}}}
```

    $\mathbf{A}_{\{\alpha\}} \setminus \mathbf{B}_{\{\alpha\}}$

2.10 Macros for Tuples

1.
```
\cTupleTyL
  {\alpha_1
   \times
   \cdots
   \times
   \alpha_n}
```

    $(\alpha_1 \times \cdots \times \alpha_n)$

2.
```
\cTupleL
  {\mathbf{A}^{1}_{\cB}
   ,
   \ldots
   ,
   \mathbf{A}^{n}_{\cB}}
```

    $(\mathbf{A}^1_o, \ldots, \mathbf{A}^n_o)$

3. `\cFstPC {\alpha} {\beta}`

    $\mathsf{fst}_{(\alpha\times\beta)\to\alpha}$

4. `\cSndPC {\alpha} {\beta}`

    $\mathsf{snd}_{(\alpha\times\beta)\to\beta}$

2.11 Macros for Functions

1. `\cDomPC {\alpha} {\beta}`

   $\mathsf{dom}_{(\alpha\to\beta)\to\{\alpha\}}$

2. `\cRanPC {\alpha} {\beta}`

   $\mathsf{ran}_{(\alpha\to\beta)\to\{\beta\}}$

3. `\cSubfuneqPC {\alpha} {\beta}`

   $\sqsubseteq_{(\alpha\to\beta)\to(\alpha\to\beta)\to o}$

4. `\cFunCompPC {\alpha} {\beta} {\gamma}`

   $\circ_{(\alpha\to\beta)\to(\beta\to\gamma)\to(\alpha\to\gamma)}$

5. `\cFunComp`
   `{\mathbf{F}_{\cFunTyX {\alpha} {\beta}}}`
   `{\mathbf{G}_{\cFunTyX {\beta} {\gamma}}}`

   $(\mathbf{F}_{\alpha\to\beta} \circ \mathbf{G}_{\beta\to\gamma})$

6. `\cFunCompX`
   `{\mathbf{F}_{\cFunTyX {\alpha} {\beta}}}`
   `{\mathbf{G}_{\cFunTyX {\beta} {\gamma}}}`

   $\mathbf{F}_{\alpha\to\beta} \circ \mathbf{G}_{\beta\to\gamma}$

7. `\cRestrictPC {\alpha} {\beta}`

   $|_{(\alpha\to\beta)\to\{\alpha\}\to(\alpha\to\beta)}$

8. `\cRestrict`
   `{\mathbf{F}_{\cFunTyX {\alpha} {\beta}}}`
   `{\mathbf{A}_{\cSetTy {\alpha}}}`

   $(\mathbf{F}_{\alpha\to\beta}|_{\mathbf{A}_{\{\alpha\}}})$

9. `\cRestrictX`
   `{\mathbf{F}_{\cFunTyX {\alpha} {\beta}}}`
   `{\mathbf{A}_{\cSetTy {\alpha}}}`

   $\mathbf{F}_{\alpha\to\beta}|_{\mathbf{A}_{\{\alpha\}}}$

2.12 Macros for Miscellaneous Notation

1. \cTotal
```
{\mathbf{F}_{\cFunTyX {\alpha} {\beta}}}
```
$\mathsf{TOTAL}(\mathbf{F}_{\alpha\rightarrow\beta})$

2. \cTotalB
```
{\mathbf{F}_{\cFunTyBX {\alpha} {\beta} {\gamma}}}
```
$\mathsf{TOTAL2}(\mathbf{F}_{\alpha\rightarrow\beta\rightarrow\gamma})$

3. \cSurj
```
{\mathbf{F}_{\cFunTyX {\alpha} {\beta}}}
```
$\mathsf{SURJ}(\mathbf{F}_{\alpha\rightarrow\beta})$

4. \cSurjB
```
{\mathbf{F}_{\cFunTyBX {\alpha} {\beta} {\gamma}}}
```
$\mathsf{SURJ2}(\mathbf{F}_{\alpha\rightarrow\beta\rightarrow\gamma})$

5. \cInj
```
{\mathbf{F}_{\cFunTyX {\alpha} {\beta}}}
```
$\mathsf{INJ}(\mathbf{F}_{\alpha\rightarrow\beta})$

6. \cInjB
```
{\mathbf{F}_{\cFunTyBX {\alpha} {\beta} {\gamma}}}
```
$\mathsf{INJ2}(\mathbf{F}_{\alpha\rightarrow\beta\rightarrow\gamma})$

7. \cBij
```
{\mathbf{F}_{\cFunTyX {\alpha} {\beta}}}
```
$\mathsf{BIJ}(\mathbf{F}_{\alpha\rightarrow\beta})$

8. \cDistinctL
```
{\mathbf{A}^{1}_{\alpha}
 ,
 \ldots
 ,
 \mathbf{A}^{n}_{\alpha}}
```
$\mathsf{DISTINCT}(\mathbf{A}^1_\alpha, \ldots, \mathbf{A}^n_\alpha)$

2.13 Macros for Quasitypes

1. ```
   \cFunAbsQTy
     {\mathbf{x}}
     {\mathbf{Q}_{\cSetTy {\alpha}}}
     {\mathbf{B}_\beta}
   ```

   $(\lambda\, \mathbf{x} : \mathbf{Q}_{\{\alpha\}} \,.\, \mathbf{B}_\beta)$

2. ```
   \cFunAbsQTyX
     {\mathbf{x}}
     {\mathbf{Q}_{\cSetTy {\alpha}}}
     {\mathbf{B}_\beta}
   ```

   $\lambda\, \mathbf{x} : \mathbf{Q}_{\{\alpha\}} \,.\, \mathbf{B}_\beta$

3. ```
   \cForallQTy
     {\mathbf{x}}
     {\mathbf{Q}_{\cSetTy {\alpha}}}
     {\mathbf{B}_\cB}
   ```

   $(\forall\, \mathbf{x} : \mathbf{Q}_{\{\alpha\}} \,.\, \mathbf{B}_o)$

4. ```
   \cForallQTyX
     {\mathbf{x}}
     {\mathbf{Q}_{\cSetTy {\alpha}}}
     {\mathbf{B}_\cB}
   ```

   $\forall\, \mathbf{x} : \mathbf{Q}_{\{\alpha\}} \,.\, \mathbf{B}_o$

5. ```
   \cForallQTyB
     {\mathbf{x}}
     {\mathbf{Q}_{\cSetTy {\alpha}}}
     {\mathbf{y}}
     {\mathbf{R}_{\cSetTy {\beta}}}
     {\mathbf{B}_\cB}
   ```

   $(\forall\, \mathbf{x} : \mathbf{Q}_{\{\alpha\}}, \mathbf{y} : \mathbf{R}_{\{\beta\}} \,.\, \mathbf{B}_o)$

6. ```
   \cForallQTyBX
     {\mathbf{x}}
     {\mathbf{Q}_{\cSetTy {\alpha}}}
     {\mathbf{y}}
     {\mathbf{R}_{\cSetTy {\beta}}}
   ```

```
{\mathbf{B}_\cB}
```

$\forall \mathbf{x} : \mathbf{Q}_{\{\alpha\}}, \mathbf{y} : \mathbf{R}_{\{\beta\}} \mathrel{.} \mathbf{B}_o$

7.
```
\cForsomeQTy
  {\mathbf{x}}
  {\mathbf{Q}_{\cSetTy {\alpha}}}
  {\mathbf{B}_\cB}
```

$(\exists \mathbf{x} : \mathbf{Q}_{\{\alpha\}} \mathrel{.} \mathbf{B}_o)$

8.
```
\cForsomeQTyX
  {\mathbf{x}}
  {\mathbf{Q}_{\cSetTy {\alpha}}}
  {\mathbf{B}_\cB}
```

$\exists \mathbf{x} : \mathbf{Q}_{\{\alpha\}} \mathrel{.} \mathbf{B}_o$

9.
```
\cForsomeQTyB
  {\mathbf{x}}
  {\mathbf{Q}_{\cSetTy {\alpha}}}
  {\mathbf{y}}
  {\mathbf{R}_{\cSetTy {\beta}}}
  {\mathbf{B}_\cB}
```

$(\exists \mathbf{x} : \mathbf{Q}_{\{\alpha\}}, \mathbf{y} : \mathbf{R}_{\{\beta\}} \mathrel{.} \mathbf{B}_o)$

10.
```
\cForsomeQTyBX
  {\mathbf{x}}
  {\mathbf{Q}_{\cSetTy {\alpha}}}
  {\mathbf{y}}
  {\mathbf{R}_{\cSetTy {\beta}}}
  {\mathbf{B}_\cB}
```

$\exists \mathbf{x} : \mathbf{Q}_{\{\alpha\}}, \mathbf{y} : \mathbf{R}_{\{\beta\}} \mathrel{.} \mathbf{B}_o$

11.
```
\cDefDesQTy
  {\mathbf{x}}
  {\mathbf{Q}_{\cSetTy {\alpha}}}
  {\mathbf{B}_\cB}
```

$(\mathrm{I}\, \mathbf{x} : \mathbf{Q}_{\{\alpha\}} \mathrel{.} \mathbf{B}_o)$

12.
```
\cDefDesQTyX
  {\mathbf{x}}
```

```
{\mathbf{Q}_{\cSetTy {\alpha}}}
{\mathbf{B}_\cB}
```

$\mathrm{I}\,\mathbf{x} : \mathbf{Q}_{\{\alpha\}} \,.\, \mathbf{B}_o$

13. `\cIsDefInQTy`
```
{\mathbf{A}_\alpha}
{\mathbf{Q}_{\cSetTy {\alpha}}}
```

$(\mathbf{A}_\alpha \downarrow \mathbf{Q}_{\{\alpha\}})$

14. `\cIsDefInQTyX`
```
{\mathbf{A}_\alpha}
{\mathbf{Q}_{\cSetTy {\alpha}}}
```

$\mathbf{A}_\alpha \downarrow \mathbf{Q}_{\{\alpha\}}$

15. `\cIsUndInQTy`
```
{\mathbf{A}_\alpha}
{\mathbf{Q}_{\cSetTy {\alpha}}}
```

$(\mathbf{A}_\alpha \uparrow \mathbf{Q}_{\{\alpha\}})$

16. `\cIsUndefInQTyX`
```
{\mathbf{A}_\alpha}
{\mathbf{Q}_{\cSetTy {\alpha}}}
```

$\mathbf{A}_\alpha \uparrow \mathbf{Q}_{\{\alpha\}}$

17. `\cFunQTyPC {\alpha} {\beta}`

$\rightarrow_{\{\alpha\}\rightarrow\{\beta\}\rightarrow\{\alpha\rightarrow\beta\}}$

18. `\cFunQTy`
```
{\mathbf{Q}_{\cSetTy {\alpha}}}
{\mathbf{R}_{\cSetTy {\beta}}}
```

$(\mathbf{Q}_{\{\alpha\}} \rightarrow \mathbf{R}_{\{\beta\}})$

19. `\cFunQTyX`
```
{\mathbf{Q}_{\cSetTy {\alpha}}}
{\mathbf{R}_{\cSetTy {\beta}}}
```

$\mathbf{Q}_{\{\alpha\}} \rightarrow \mathbf{R}_{\{\beta\}}$

20. `\cFunQTyB`
```
{\mathbf{Q}_{\cSetTy {\alpha}}}
```

```
{\mathbf{R}_{\cSetTy {\beta}}}
{\mathbf{S}_{\cSetTy {\gamma}}}
```

$(\mathbf{Q}_{\{\alpha\}} \to \mathbf{R}_{\{\beta\}} \to \mathbf{S}_{\{\gamma\}})$

21. `\cFunQTyBX`
```
{\mathbf{Q}_{\cSetTy {\alpha}}}
{\mathbf{R}_{\cSetTy {\beta}}}
{\mathbf{S}_{\cSetTy {\gamma}}}
```

$\mathbf{Q}_{\{\alpha\}} \to \mathbf{R}_{\{\beta\}} \to \mathbf{S}_{\{\gamma\}}$

22. `\cFunQTyC`
```
{\mathbf{Q}_{\cSetTy {\alpha}}}
{\mathbf{R}_{\cSetTy {\beta}}}
{\mathbf{S}_{\cSetTy {\gamma}}}
{\mathbf{T}_{\cSetTy {\delta}}}
```

$(\mathbf{Q}_{\{\alpha\}} \to \mathbf{R}_{\{\beta\}} \to \mathbf{S}_{\{\gamma\}} \to \mathbf{T}_{\{\delta\}})$

23. `\cFunQTyCX`
```
{\mathbf{Q}_{\cSetTy {\alpha}}}
{\mathbf{R}_{\cSetTy {\beta}}}
{\mathbf{S}_{\cSetTy {\gamma}}}
{\mathbf{T}_{\cSetTy {\delta}}}
```

$\mathbf{Q}_{\{\alpha\}} \to \mathbf{R}_{\{\beta\}} \to \mathbf{S}_{\{\gamma\}} \to \mathbf{T}_{\{\delta\}}$

24. `\cProdQTyPC {\alpha} {\beta}`

$\times_{\{\alpha\} \to \{\beta\} \to \{\alpha \times \beta\}}$

25. `\cProdQTy`
```
{\mathbf{Q}_{\cSetTy {\alpha}}}
{\mathbf{R}_{\cSetTy {\beta}}}
```

$(\mathbf{Q}_{\{\alpha\}} \times \mathbf{R}_{\{\beta\}})$

26. `\cProdQTyX`
```
{\mathbf{Q}_{\cSetTy {\alpha}}}
{\mathbf{R}_{\cSetTy {\beta}}}
```

$\mathbf{Q}_{\{\alpha\}} \times \mathbf{R}_{\{\beta\}}$

27. `\cProdQTyB`
```
{\mathbf{Q}_{\cSetTy {\alpha}}}
```

```
{\mathbf{R}_{\cSetTy {\beta}}}
{\mathbf{S}_{\cSetTy {\gamma}}}
```

$(\mathbf{Q}_{\{\alpha\}} \times \mathbf{R}_{\{\beta\}} \times \mathbf{S}_{\{\gamma\}})$

28. `\cProdQTyBX`
```
{\mathbf{Q}_{\cSetTy {\alpha}}}
{\mathbf{R}_{\cSetTy {\beta}}}
{\mathbf{S}_{\cSetTy {\gamma}}}
```

$\mathbf{Q}_{\{\alpha\}} \times \mathbf{R}_{\{\beta\}} \times \mathbf{S}_{\{\gamma\}}$

29. `\cProdQTyC`
```
{\mathbf{Q}_{\cSetTy {\alpha}}}
{\mathbf{R}_{\cSetTy {\beta}}}
{\mathbf{S}_{\cSetTy {\gamma}}}
{\mathbf{T}_{\cSetTy {\delta}}}
```

$(\mathbf{Q}_{\{\alpha\}} \times \mathbf{R}_{\{\beta\}} \times \mathbf{S}_{\{\gamma\}} \times \mathbf{T}_{\{\delta\}})$

30. `\cProdQTyCX`
```
{\mathbf{Q}_{\cSetTy {\alpha}}}
{\mathbf{R}_{\cSetTy {\beta}}}
{\mathbf{S}_{\cSetTy {\gamma}}}
{\mathbf{T}_{\cSetTy {\delta}}}
```

$\mathbf{Q}_{\{\alpha\}} \times \mathbf{R}_{\{\beta\}} \times \mathbf{S}_{\{\gamma\}} \times \mathbf{T}_{\{\delta\}}$

31. `\cSetQTy`
```
{\mathbf{Q}_{\cSetTy {\alpha}}}
```

$\mathcal{P}(\mathbf{Q}_{\{\alpha\}})$

32. `\cTotalOn`
```
{\mathbf{F}_{\cFunTyX {\alpha} {\beta}}}
{\mathbf{Q}_{\cSetTy {\alpha}}}
{\mathbf{R}_{\cSetTy {\beta}}}
```

$\textsf{TOTAL-ON}(\mathbf{F}_{\alpha\rightarrow\beta}, \mathbf{Q}_{\{\alpha\}}, \mathbf{R}_{\{\beta\}})$

33. `\cTotalOnB`
```
{\mathbf{F}_{\cFunTyBX {\alpha} {\beta} {\gamma}}}
{\mathbf{Q}_{\cSetTy {\alpha}}}
{\mathbf{R}_{\cSetTy {\beta}}}
{\mathbf{S}_{\cSetTy {\gamma}}}
```

TOTAL-ON2($\mathbf{F}_{\alpha\to\beta\to\gamma}, \mathbf{Q}_{\{\alpha\}}, \mathbf{R}_{\{\beta\}}, \mathbf{S}_{\{\gamma\}}$)

34. \cSurjOn
```
{\mathbf{F}_{\cFunTyX {\alpha} {\beta}}}
{\mathbf{Q}_{\cSetTy {\alpha}}}
{\mathbf{R}_{\cSetTy {\beta}}}
```
SURJ-ON($\mathbf{F}_{\alpha\to\beta}, \mathbf{Q}_{\{\alpha\}}, \mathbf{R}_{\{\beta\}}$)

35. \cSurjOnB
```
{\mathbf{F}_{\cFunTyBX {\alpha} {\beta} {\gamma}}}
{\mathbf{Q}_{\cSetTy {\alpha}}}
{\mathbf{R}_{\cSetTy {\beta}}}
{\mathbf{S}_{\cSetTy {\gamma}}}
```
SURJ-ON2($\mathbf{F}_{\alpha\to\beta\to\gamma}, \mathbf{Q}_{\{\alpha\}}, \mathbf{R}_{\{\beta\}}, \mathbf{S}_{\{\gamma\}}$)

36. \cInjOn
```
{\mathbf{F}_{\cFunTyX {\alpha} {\beta}}}
{\mathbf{Q}_{\cSetTy {\alpha}}}
```
INJ-ON($\mathbf{F}_{\alpha\to\beta}, \mathbf{Q}_{\{\alpha\}}$)

37. \cInjOnB
```
{\mathbf{F}_{\cFunTyBX {\alpha} {\beta} {\gamma}}}
{\mathbf{Q}_{\cSetTy {\alpha}}}
{\mathbf{R}_{\cSetTy {\beta}}}
```
INJ-ON2($\mathbf{F}_{\alpha\to\beta\to\gamma}, \mathbf{Q}_{\{\alpha\}}, \mathbf{R}_{\{\beta\}}$)

38. \cBijOn
```
{\mathbf{F}_{\cFunTyX {\alpha} {\beta}}}
{\mathbf{Q}_{\cSetTy {\alpha}}}
{\mathbf{R}_{\cSetTy {\beta}}}
```
BIJ-ON($\mathbf{F}_{\alpha\to\beta}, \mathbf{Q}_{\{\alpha\}}, \mathbf{R}_{\{\beta\}}$)

39. \cInf {\mathbf{Q}_{\cSetTy {\alpha}}}

INF($\mathbf{Q}_{\{\alpha\}}$)

40. \cFin {\mathbf{Q}_{\cSetTy {\alpha}}}

FIN($\mathbf{Q}_{\{\alpha\}}$)

41. \cCount {\mathbf{Q}_{\cSetTy {\alpha}}}

COUNT($\mathbf{Q}_{\{\alpha\}}$)

2.14 Macros for Dependent Types

1. `\cPiTy {\alpha} {\beta}`

   $\{\alpha\} \to (\alpha \to \{\beta\}) \to \{\alpha \to \beta\}$

2. `\cPiPC {\alpha} {\beta}`

   $\Pi_{\{\alpha\} \to (\alpha \to \{\beta\}) \to \{\alpha \to \beta\}}$

3. ```
   \cPi
     {\mathbf{x}}
     {\mathbf{Q}_{\cSetTy {\alpha}}}
     {\mathbf{R}_{\cSetTy {\beta}}}
   ```

   $(\Pi\, \mathbf{x} : \mathbf{Q}_{\{\alpha\}} \,.\, \mathbf{R}_{\{\beta\}})$

4. ```
   \cPiX
     {\mathbf{x}}
     {\mathbf{Q}_{\cSetTy {\alpha}}}
     {\mathbf{R}_{\cSetTy {\beta}}}
   ```

   $\Pi\, \mathbf{x} : \mathbf{Q}_{\{\alpha\}} \,.\, \mathbf{R}_{\{\beta\}}$

5. `\cSigmaTy {\alpha} {\beta}`

   $\{\alpha\} \to (\alpha \to \{\beta\}) \to \{\alpha \times \beta\}$

6. `\cSigmaPC {\alpha} {\beta}`

   $\Sigma_{\{\alpha\} \to (\alpha \to \{\beta\}) \to \{\alpha \times \beta\}}$

7. ```
   \cSigma
     {\mathbf{x}}
     {\mathbf{Q}_{\cSetTy {\alpha}}}
     {\mathbf{R}_{\cSetTy {\beta}}}
   ```

   $(\Sigma\, \mathbf{x} : \mathbf{Q}_{\{\alpha\}} \,.\, \mathbf{R}_{\{\beta\}})$

8. ```
   \cSigmaX
     {\mathbf{x}}
     {\mathbf{Q}_{\cSetTy {\alpha}}}
     {\mathbf{R}_{\cSetTy {\beta}}}
   ```

   $\Sigma\, \mathbf{x} : \mathbf{Q}_{\{\alpha\}} \,.\, \mathbf{R}_{\{\beta\}}$

2.15 Macros for Sequences

1. `\cSequencesPC {\alpha} {\beta}`

   $\mathsf{sequences}_{\{\alpha \to \beta\}}$

2. `\cSeqQTy {\beta}`

   $\langle\!\langle \beta \rangle\!\rangle$

3. `\cStreamsPC {\alpha} {\beta}`

   $\mathsf{streams}_{\{\alpha \to \beta\}}$

4. `\cSeqInfQTy {\beta}`

   $\langle \beta \rangle$

5. `\cListsPC {\alpha} {\beta}`

   $\mathsf{lists}_{\{\alpha \to \beta\}}$

6. `\cSeqFinQTy {\beta}`

   $[\beta]$

7. `\cConsPC {\alpha} {\beta}`

   $\mathsf{cons}_{\beta \to (\alpha \to \beta) \to (\alpha \to \beta)}$

8. ```
   \cCons
     {\mathbf{A}_\beta}
     {\mathbf{B}_{\cFunTyX {\alpha} {\beta}}}
   ```

   $(\mathbf{A}_\beta :: \mathbf{B}_{\alpha \to \beta})$

9. ```
   \cConsX
     {\mathbf{A}_\beta}
     {\mathbf{B}_{\cFunTyX {\alpha} {\beta}}}
   ```

   $\mathbf{A}_\beta :: \mathbf{B}_{\alpha \to \beta}$

10. `\cHdPC {\alpha} {\beta}`

    $\mathsf{hd}_{(\alpha \to \beta) \to \beta}$

11. `\cTlPC {\alpha} {\beta}`

    $\mathsf{tl}_{(\alpha \to \beta) \to (\alpha \to \beta)}$

12. \cNilPC {\alpha} {\beta}

    $\mathsf{nil}_{\alpha\to\beta}$

13. \cEmpListPC {\alpha} {\beta}

    $[\,]_{\alpha\to\beta}$

14. \cListL

    ```
    {\mathbf{A}^{1}_{\beta}
    ,
    \ldots
    ,
    \mathbf{A}^{n}_{\beta}}
    ```

    $[\mathbf{A}^1_\beta, \ldots, \mathbf{A}^n_\beta]$

15. \cLenPC {\alpha} {\beta}

    $\mathsf{len}_{(\alpha\to\beta)\to\alpha}$

16. \cLen {\mathbf{A}_{\cFunTyX {\alpha} {\beta}}}

    $|\mathbf{A}_{\alpha\to\beta}|$

17. \cAppendPC {\alpha} {\beta}

    $++_{(\alpha\to\beta)\to(\alpha\to\beta)\to(\alpha\to\beta)}$

18. \cAppend

    ```
    {\mathbf{A}_{\cFunTyX {\alpha} {\beta}}}
    {\mathbf{B}_{\cFunTyX {\alpha} {\beta}}}
    ```

    $(\mathbf{A}_{\alpha\to\beta} ++ \mathbf{B}_{\alpha\to\beta})$

19. \cAppendX

    ```
    {\mathbf{A}_{\cFunTyX {\alpha} {\beta}}}
    {\mathbf{B}_{\cFunTyX {\alpha} {\beta}}}
    ```

    $\mathbf{A}_{\alpha\to\beta} ++ \mathbf{B}_{\alpha\to\beta}$

20. \cNlistsPC {\alpha} {\beta}

    $\mathsf{nlists}_{\alpha\to\{\alpha\to\beta\}}$

21. \cSeqNFinQTy {\beta} {\mathbf{N}_\alpha}

    $[\beta]^{\mathbf{N}_\alpha}$

2.16 Macros for Reals

1. `\cRecip {\mathbf{A}_R}`

   $\left(\mathbf{A}_R{}^{-1}\right)$

2. `\cRecipX {\mathbf{A}_R}`

   $\mathbf{A}_R{}^{-1}$

3. `\cFrac {\mathbf{A}_R} {\mathbf{B}_R}`

   $\left(\frac{\mathbf{A}_R}{\mathbf{B}_R}\right)$

4. `\cFracX {\mathbf{A}_R} {\mathbf{B}_R}`

   $\frac{\mathbf{A}_R}{\mathbf{B}_R}$

5. `\cAbs {\mathbf{A}_R}`

   $|\mathbf{A}_R|$

6. `\cSqrt {\mathbf{A}_R}`

   $\sqrt{\mathbf{A}_R}$

7. `\cNorm {\mathbf{A}_{\cFunTyX {R} {R}}}`

   $\|\mathbf{A}_{R \to R}\|$

8. `\cSum`
   ```
   {\mathbf{i}}
   {\mathbf{M}_R}
   {\mathbf{N}_R} {\mathbf{A}_R}
   ```

   $\left(\sum\limits_{\mathbf{i}=\mathbf{M}_R}^{\mathbf{N}_R} \mathbf{A}_R\right)$

9. `\cSumX`
   ```
   {\mathbf{i}}
   {\mathbf{M}_R}
   {\mathbf{N}_R} {\mathbf{A}_R}
   ```

   $\sum\limits_{\mathbf{i}=\mathbf{M}_R}^{\mathbf{N}_R} \mathbf{A}_R$

10. `\cProd`
    ```
    {\mathbf{i}}
    {\mathbf{M}_R}
    ```

```
{\mathbf{N}_R} {\mathbf{A}_R}
```

$$\left(\prod_{\mathbf{i}=\mathbf{M}_R}^{\mathbf{N}_R} \mathbf{A}_R\right)$$

11. `\cProdX`

```
{\mathbf{i}}
{\mathbf{M}_R}
{\mathbf{N}_R} {\mathbf{A}_R}
```

$$\prod_{\mathbf{i}=\mathbf{M}_R}^{\mathbf{N}_R} \mathbf{A}_R$$

12. `\cLim`

```
{\mathbf{x}}
{\mathbf{A}_R}
{\mathbf{B}_R}
```

$$\left(\lim_{\mathbf{x}\to\mathbf{A}_R} \mathbf{B}_R\right)$$

13. `\cLimX`

```
{\mathbf{x}}
{\mathbf{A}_R}
{\mathbf{B}_R}
```

$$\lim_{\mathbf{x}\to\mathbf{A}_R} \mathbf{B}_R$$

14. `\cRightLim`

```
{\mathbf{x}}
{\mathbf{A}_R}
{\mathbf{B}_R}
```

$$\left(\lim_{\mathbf{x}\to\mathbf{A}_R{}^+} \mathbf{B}_R\right)$$

15. `\cRightLimX`

```
{\mathbf{x}}
{\mathbf{A}_R}
{\mathbf{B}_R}
```

$$\lim_{\mathbf{x}\to\mathbf{A}_R{}^+} \mathbf{B}_R$$

16. `\cLeftLim`

```
{\mathbf{x}}
```

```
{\mathbf{A}_R}
{\mathbf{B}_R}
```

$\left(\lim_{\mathbf{x} \to {\mathbf{A}_R}^-} \mathbf{B}_R\right)$

17. `\cLeftLimX`

```
{\mathbf{x}}
{\mathbf{A}_R}
{\mathbf{B}_R}
```

$\lim_{\mathbf{x} \to {\mathbf{A}_R}^-} \mathbf{B}_R$

18. `\cLimSeq`

```
{\mathbf{n}}
{\mathbf{B}_R}
```

$\left(\lim_{\mathbf{n} \to \infty} \mathbf{B}_R\right)$

19. `\cLimSeqX`

```
{\mathbf{n}}
{\mathbf{B}_R}
```

$\lim_{\mathbf{n} \to \infty} \mathbf{B}_R$

20. `\cIntegral`

```
{\mathbf{A}_R}
{\mathbf{B}_R}
{\mathbf{C}_R}
{\mathbf{x}}
```

$\left(\int_{\mathbf{A}_R}^{\mathbf{B}_R} \mathbf{C}_R \, d\mathbf{x}\right)$

21. `\cIntegralX`

```
{\mathbf{A}_R}
{\mathbf{B}_R}
{\mathbf{C}_R}
{\mathbf{x}}
```

$\int_{\mathbf{A}_R}^{\mathbf{B}_R} \mathbf{C}_R \, d\mathbf{x}$

3 Environments

In this section, an example of each LaTeX environment in

`alonzo-notation.tex`

is presented consisting of the LaTeX source code for the environment followed by the output it produces.

3.1 Theory Definition

```
\begin{theory-def}
{Monoids}
{$\mName{MON}$.}
{$S$.}
{$\cdot_{\cFunTyBX {S} {S} {S}}, e_S$.}
{
\be

  \item $\cForallX {x,y,z} {S} {\cEqX {\cBinX {x} {\cdot} {\cBin {y}
       {\cdot} {z}}} {\cBinX {\cBin {x} {\cdot} {y}} {\cdot} {z}}}$
      \hfill (associativity).

  \item $\cForallX {x} {S} {\cEqX {\cBinX {e} {\cdot} {x}} {x}}$ \
      hfill (left identity).

  \item $\cForallX {x} {S} {\cEqX {\cBinX {x} {\cdot} {e}} {x}}$ \
      hfill (right identity).

\ee
}
\end{theory-def}
```

Theory Definition 3.1 (Monoids)

Name: MON.

Base types: S.

Constants: $\cdot_{S \rightarrow S \rightarrow S}, e_S$.

Axioms:

1. $\forall\, x, y, z : S \,.\, x \cdot (y \cdot z) = (x \cdot y) \cdot z$ (associativity).
2. $\forall\, x : S \,.\, e \cdot x = x$ (left identity).
3. $\forall\, x : S \,.\, x \cdot e = x$ (right identity).

3.2 Theory Extension

```
\begin{theory-ext}
{Groups}
{$\mName{GRP}$.}
{$\mName{MON}$.}
{(none).}
{${\recip{\cdot}}_{\cFunTyX {S} {S}}$.}
{
\be \setcounter{enumi}{3}

  \item $\cForallX {x} {S} {\cEqX {\cBinX {\recip{x}} {\cdot} {x}} {
      e}}$ \hfill (left inverse).

  \item $\cForallX {x} {S} {\cEqX {\cBinX {x} {\cdot} {\recip{x}}} {
      e}}$ \hfill (right inverse).

\ee
}
\end{theory-ext}
```

Theory Extension 3.2 (Groups)

Name: GRP.

Extends MON.

New base types: (none).

New constants: ${\cdot^{-1}}_{S \rightarrow S}$.

New axioms:

4. $\forall\, x : S \,.\, x^{-1} \cdot x = e$ (left inverse).
5. $\forall\, x : S \,.\, x \cdot x^{-1} = e$ (right inverse).

3.3 Inductive Type Theory Extension

```
\begin{ind-type-theory-ext}
{Peano Arithmetic}
{$\textsf{PA-ALT}$.}
{$\mName{MIN}$.}
{$N$.}
{
```

```
\be

  \item $0_N$.

  \item $S_{\cFunTyX {N} {N}}$.

\ee
}
\end{ind-type-theory-ext}
```

Inductive Type Theory Extension 3.3 (Peano Arithmetic)

Name: PA-ALT.

Extends MIN.

New base type: N.

Constructors:

1. 0_N.
2. $S_{N \to N}$.

3.4 Development Definition

```
\begin{dev-def}
{$\mName{PA}$ Development 1}
{\textsf{PA-dev-1}.}
{$\mName{PA}$.}
{
\bi

  \item[] $\mName{Def1}$: $\cEqX {1_N} {\cFunAppX {S} {0}}$ \hfill (
      number one).

  \item[] $\mName{Def2}$: $\cEqX {P_{\cFunTyX {N} {N}}} {\cFunAbsX {
      x} {N} {\cDefDesX {y} {N} {\cEqX {\cFunAppX {S} {y}} {x}}}}$ \
      hfill (predecessor function).

  \item[] $\mName{Def3}$: $\cEqX {+_{\cFunTyBX {N} {N} {N}}} {}$\\
     \hspace*{2ex}${\cDefDesX {f} {\cFunTyBX {N} {N} {N}} {\cForallX
         {x, y} {N} {\cAndX {\cEqX {\cFunAppBX {f} {x} {0}} {x}} {\
         cEqX {\cFunAppBX {f} {x} {\cFunApp {S} {y}}} {\cFunAppX {S}
         {\cFunAppB {f} {x} {y}}}}}}}}$\\ \phantom{x} \hfill (addition
          function).
```

```
  \item[] $\mName{Def4}$: $\cEqX {*_{\cFunTyBX {N} {N} {N}}} {}$\\
    \hspace*{2ex}${\cDefDesX {f} {\cFunTyBX {N} {N} {N}} {\cForallX
        {x, y} {N} {\cAndX {\cEqX {\cFunAppBX {f} {x} {0}} {0}} {\
        cEqX {\cFunAppBX {f} {x} {\cFunApp {S} {y}}} {\cBinX {\
        cFunAppB {f} {x} {y}} {+} {x}}}}}}}$\\ \phantom{x} \hfill (
        multiplication function).

  \item[] $\mName{Thm1}$: $\cForallX {x} {N} {\cIfX {\cEqX {x} {0}}
      {\cIsUndefX {\cFunApp {P} {x}}} {\cIsDefX {\cFunApp {P} {x}}}}
      $ \hfill ($P$ is almost total).

  \item[] $\mName{Thm2}$: $\cForallX {x} {N} {\cEqX {\cFunAppX {P}
      {\cFunApp {S} {x}}} {x}}$ \hfill ($P$ is a left inverse of $S$
      ).

  \item[] $\mName{Thm3}$: $\cTotalB {+}$. \hfill ($+$ is total).

  \item[] $\mName{Thm4}$: $\cTotalB {*}$. \hfill ($*$ is total).

\ei
}
\end{dev-def}
```

Development Definition 3.4 (PA **Development 1**)

Name: PA-dev-1.

Bottom theory: PA.

Definitions and theorems:

Def1: $1_N = S\,0$ (number one).

Def2: $P_{N \to N} = \lambda\, x : N \,.\, \mathrm{I}\, y : N \,.\, S\, y = x$ (predecessor function).

Def3: $+_{N \to N \to N} =$
$\mathrm{I}\, f : N \to N \to N \,.\, \forall\, x, y : N \,.\, f\, x\, 0 = x \wedge f\, x\, (S\, y) = S\, (f\, x\, y)$
(addition function).

Def4: $*_{N \to N \to N} =$
$\mathrm{I}\, f : N \to N \to N \,.\, \forall\, x, y : N \,.\, f\, x\, 0 = 0 \wedge f\, x\, (S\, y) = (f\, x\, y) + x$
(multiplication function).

Thm1: $\forall\, x : N \,.\, x = 0 \mapsto (P\, x)\!\uparrow \mid (P\, x)\!\downarrow$ (P is almost total).

Thm2: $\forall\, x : N\, .\, P\,(S\,x) = x$ \hfill (P is a left inverse of S).

Thm3: TOTAL2(+). (+ is total).

Thm4: TOTAL2($*$). ($*$ is total).

3.5 Development Extension

```
\begin{dev-ext}
{$\mName{PA}$ Development 2}
{\textsf{PA-dev-2}.}
{\textsf{PA-dev-1}.}
{
\bi

  \item[] $\mName{Thm5}$: $\cForallX {x,y,z} {N} {\cEqX {\cBinX {\
      cBin {x} {+} {y}} {+} {z}} {\cBinX {x} {+} {\cBin {y} {+} {z
      }}}}$ \hfill (associativity of $+$).

  \item[] $\mName{Thm6}$: $\cForallX {x} {N} {\cBinBX {\cBinX {0}
      {+} {x}} {=} {x} {=} {\cBinX {x} {+} {0}}}$ \hfill ($0$ is a $
      +$-identity).

  \item[] $\mName{Thm7}$: $\cForallX {x,y} {N} {\cEqX {\cBinX {x}
      {+} {y}} {\cBinX {y} {+} {x}}}$ \hfill (commutativity of $+$).

  \item[] $\mName{Thm8}$: $\cForallX {x,y,z} {N} {\cEqX {\cBinX {\
      cBin {x} {*} {y}} {*} {z}} {\cBinX {x} {*} {\cBin {y} {*} {z
      }}}}$ \hfill (associativity of $*$).

  \item[] $\mName{Thm9}$: $\cForallX {x} {N} {\cBinBX {\cBinX {1}
      {*} {x}} {=} {x} {=} {\cBinX {x} {*} {1}}}$ \hfill ($1$ is a $
      *$-identity).

  \item[] $\mName{Thm10}$: $\cForallX {x,y} {N} {\cEqX {\cBinX {x}
      {*} {y}} {\cBinX {y} {*} {x}}}$ \hfill (commutativity of $*$).

  \item[] $\mName{Thm11}$: $\cForallX {x,y,z} {N} {\cEqX {\cBinX {x}
       {*} {\cBin {y} {+} {z}}} {\cBinX {\cBin {x} {*} {y}} {+} {\
      cBin {x} {*} {z}}}}$\\ \phantom{x} \hfill (left distributivity
      ).

  \item[] $\mName{Thm12}$: $\cForallX {x,y,z} {N} {\cEqX {\cBinX {\
      cBin {x} {+} {y}} {*} {z}} {\cBinX {\cBin {x} {*} {z}} {+} {\
```

```
      cBin {y} {*} {z}}}}$\\ \phantom{x} \hfill (right
      distributivity).

  \item[] $\mName{Thm13}$: $\cForallX {x} {N} {\cBinBX {\cBinX {0}
      {*} {x}} {=} {0} {=} {\cBinX {x} {*} {0}}}$ \hfill ($0$ is an
      annihilator).

\ei
}
\end{dev-ext}
```

Development Extension 3.5 (PA **Development 2)**

Name: PA-dev-2.

Extends PA-dev-1.

New definitions and theorems:

Thm5: $\forall x, y, z : N \,.\, (x + y) + z = x + (y + z)$ (associativity of $+$).

Thm6: $\forall x : N \,.\, 0 + x = x = x + 0$ (0 is a $+$-identity).

Thm7: $\forall x, y : N \,.\, x + y = y + x$ (commutativity of $+$).

Thm8: $\forall x, y, z : N \,.\, (x * y) * z = x * (y * z)$ (associativity of $*$).

Thm9: $\forall x : N \,.\, 1 * x = x = x * 1$ (1 is a $*$-identity).

Thm10: $\forall x, y : N \,.\, x * y = y * x$ (commutativity of $*$).

Thm11: $\forall x, y, z : N \,.\, x * (y + z) = (x * y) + (x * z)$ (left distributivity).

Thm12: $\forall x, y, z : N \,.\, (x + y) * z = (x * z) + (y * z)$ (right distributivity).

Thm13: $\forall x : N \,.\, 0 * x = 0 = x * 0$ (0 is an annihilator).

3.6 Theory Translation Definition

```
\begin{theory-trans-def}
{$\mName{MON}$ to $\mName{COF}$ with $+$}
{\textsf{MON-to-COF-$+$}.}
{$\mName{MON}$.}
{$\mName{COF}$.}
{
\be

  \item $S \mapsto R$.

\ee
}
{
\be

  \item ${\cdot_{\cFunTyBX {S} {S} {S}}} \mapsto {+_{\cFunTyBX {R} {
      R} {R}}}$.

  \item $e_S \mapsto 0_R$.

\ee
}
\end{theory-trans-def}
```

Theory Translation Definition 3.6 (MON to COF with $+$)

Name: MON-to-COF-$+$.

Source theory: MON.

Target theory: COF.

Base type mapping:

1. $S \mapsto R$.

Constant mapping:

1. $\cdot_{S \to S \to S} \mapsto +_{R \to R \to R}$.
2. $e_S \mapsto 0_R$.

3.7 Theory Translation Extension

```
\begin{theory-trans-ext}
{$\mName{GRP}$ to $\mName{COF}$ with $+$}
{\textsf{GRP-to-COF-$+$}.}
{\textsf{MON-to-COF-$+$}.}
{$\mName{GRP}$.}
{$\mName{COF}$.}
{}
{
\be \setcounter{enumi}{2}

  \item ${\recip{\cdot}}_{\cFunTyX {S} {S}} \mapsto {-}_{\cFunTyX {R
      } {R}}$.

\ee
}
\end{theory-trans-ext}
```

Theory Translation Extension 3.7 (GRP to COF with $+$)

Name: GRP-to-COF-$+$.

Extends MON-to-COF-$+$.

New source theory: GRP.

New target theory: COF.

New base type mapping:

New constant mapping:

3. ${\cdot^{-1}}_{S \rightarrow S} \mapsto -_{R \rightarrow R}$.

3.8 Development Translation Definition

```
\begin{dev-trans-def}
{$\mathsf{PA}$ to $\mathsf{COF}$ 1}
{\textsf{PA-to-COF}.}
{\textsf{PA-dev-4}.}
{\textsf{COF-dev-5}.}
{
\be
```

```
  \item $N \mapsto N_{\cSetTy {R}}$.

\ee
}
{
\be

  \item $0_N \mapsto 0_R$.

  \item $S_{\cFunTyX {N} {N}} \mapsto \cFunAbsX {x} {N_{\cSetTy {R
      }}} {\cBinX {x} {+} {1}}$.

  \item $1_N \mapsto 1_R$.

  \item $P_{\cFunTyX {N} {N}} \mapsto \cFunAbsX {x} {N_{\cSetTy {R
      }}} {\cIfX {\cNotEqX {x} {0}} {\cBinX {x} {-} {1}} {\cBotPC {R
      }}}$.

  \item $+_{\cFunTyBX {N} {N} {N}} \mapsto \cFunAbsX {x} {N_{\cSetTy
       {R}}} {\cFunAbsX {y} {N_{\cSetTy {R}}} {\cBinX {x} {+} {y}}}$
      .

  \item $*_{\cFunTyBX {N} {N} {N}} \mapsto \cFunAbsX {x} {N_{\cSetTy
       {R}}} {\cFunAbsX {y} {N_{\cSetTy {R}}} {\cBinX {x} {*} {y}}}$
      .

\ee
}
\end{dev-trans-def}
```

Development Translation Definition 3.8 (PA to COF 1)

Name: PA-to-COF.

Source development: PA-dev-4.

Target development: COF-dev-5.

Base type mapping:

1. $N \mapsto N_{\{R\}}$.

Constant mapping:

1. $0_N \mapsto 0_R$.

2. $S_{N \to N} \mapsto \lambda\, x : N_{\{R\}} \,.\, x + 1.$

3. $1_N \mapsto 1_R.$

4. $P_{N \to N} \mapsto \lambda\, x : N_{\{R\}} \,.\, x \neq 0 \mapsto x - 1 \mid \bot_R.$

5. $+_{N \to N \to N} \mapsto \lambda\, x : N_{\{R\}} \,.\, \lambda\, y : N_{\{R\}} \,.\, x + y.$

6. $*_{N \to N \to N} \mapsto \lambda\, x : N_{\{R\}} \,.\, \lambda\, y : N_{\{R\}} \,.\, x * y.$

3.9 Development Translation Extension

```
\begin{dev-trans-ext}
{$\mathsf{PA}$ to $\mathsf{COF}$ 2}
{\textsf{PA-to-COF-1}$.}
{\textsf{PA-to-COF}.}
{\textsf{PA-dev-4}.}
{\textsf{COF-dev-6}.}
{
\be

  \item[] ${\mid}_{\cFunTyBX {N} {N} {\cB}} \mapsto {\mid}_{\
      cFunTyBX {R} {R} {\cB}}$.

\ee
}
\end{dev-trans-ext}
```

Development Translation Extension 3.9 (PA to COF **2**)

Name: PA-to-COF-1.

Extends PA-to-COF.

New source development: PA-dev-4.

New target development: COF-dev-6.

New defined constant mapping:

$\mid_{N \to N \to o} \mapsto \mid_{R \to R \to o}.$

3.10 Definition Transportation

```
\begin{def-transport}
{Transport of Divides to $\mathsf{COF}$}
{\textsf{Divides-via-PA-to-COF}.}
{\textsf{PA-dev-4}.}
{\textsf{COF-dev-5}.}
{\textsf{PA-to-COF}.}
{
\bi

  \item[] $\mName{Def6}$: $\cEqX {{\mid}_{\cFunTyBX {N} {N} {\cB}}}
      {\cFunAbsX {x} {N} {\cFunAbsX {y} {N} {\cForsomeX {z} {N} {\
      cEqX {\cBinX {x} {*} {z}} {y}}}}}$ \hfill (divides).

\ei
}
{
\bi

  \item[] $\textsf{Def6-via-PA-to-COF}$:\\
     \hspace*{2ex}$\cEqX {{\mid}_{\cFunTyBX {R} {R} {\cB}}} {\
         cFunAbsX {x} {N_{\cSetTy {R}}} {\cFunAbsX {y} {N_{\cSetTy {R
         }}} {\cForsomeX {z} {N_{\cSetTy {R}}} {\cEqX {\cBinX {x} {*}
          {z}} {y}}}}}$ \hfill (divides).

\ei
}
{\textsf{COF-dev-6}.}
{\textsf{PA-to-COF-1}.}
\end{def-transport}
```

Definition Transportation 3.10 (Transport of Divides to COF)

Name: Divides-via-PA-to-COF.

Source development: PA-dev-4.

Target development: COF-dev-5.

Development morphism: PA-to-COF.

Definition:

Def6: $\mid_{N \rightarrow N \rightarrow o} = \lambda\, x : N \,.\, \lambda\, y : N \,.\, \exists\, z : N \,.\, x * z = y$ (divides).

Transported definition:

Def6-via-PA-to-COF:
$|_{R \to R \to o} = \lambda\, x : N_{\{R\}} \,.\, \lambda\, y : N_{\{R\}} \,.\, \exists\, z : N_{\{R\}} \,.\, x * z = y$ (divides).

New target development: COF-dev-6.

New development morphism: PA-to-COF-1.

3.11 Theorem Transportation

```
\begin{thm-transport}
{Transport of 0 is Top to $\mathsf{COF}$}
{\textsf{0-is-Top-via-PA-to-COF-1}.}
{\textsf{PA-dev-4}.}
{\textsf{COF-dev-6}.}
{\textsf{PA-to-COF-1}.}
{
\bi

  \item[] $\mName{Thm22}$: $\cForallX {x} {N} {\cBinX {x} {{\mid}_{\
      cFunTyBX {N} {N} {\cB}}} {0}}$ \hfill ($0$ is top).

\ei
}
{
\bi

  \item[] $\textsf{Thm22-via-PA-to-COF-1}$:
     \hspace*{2ex}$\cForallX {x} {N_{\cSetTy {R}}} {\cBinX {x} {{\mid
         }_{\cFunTyBX {R} {R} {\cB}}} {0}}$ \hfill ($0$ is top).

\ei
}
{\textsf{COF-dev-7}.}
\end{thm-transport}
```

Theorem Transportation 3.11 (Transport of 0 is Top to COF)

Name: 0-is-Top-via-PA-to-COF-1.

Source development: PA-dev-4.

Target development: COF-dev-6.

Development morphism: PA-to-COF-1.

Theorem:

Thm22: $\forall\, x : N \,.\, x \mid_{N \to N \to o} 0$ (0 is top).

Transported theorem:

Thm22-via-PA-to-COF-1: $\forall\, x : N_{\{R\}} \,.\, x \mid_{R \to R \to o} 0$ (0 is top).

New target development: COF-dev-7.

3.12 Group Transportation

```
\begin{group-transport}
{\small Transport of $\mid$ and ``0 is Top'' to $\mathsf{COF}$}
{\textsf{Divides-and-0-is-top-via-PA-to-COF}.}
{\textsf{PA-dev-4}.}
{\textsf{COF-dev-5}.}
{\textsf{PA-to-COF}.}
{
\bi

  \item[] $\mName{Def6}$: $\cEqX {{\mid}_{\cFunTyBX {N} {N} {\cB}}}
    {\cFunAbsX {x} {N} {\cFunAbsX {y} {N} {\cForsomeX {z} {N} {\cEqX
            {\cBinX {x} {*} {z}} {y}}}}}$ \hfill (divides).

  \item[] $\mName{Thm22}$: $\cForallX {x} {N} {\cBinX {x}
    {{\mid}_{\cFunTyBX {N} {N} {\cB}}} {0}}$ \hfill ($0$ is top).

\ei
}
{
\bi

  \item[] \textsf{Def6-via-PA-to-COF}:\\
   \hspace*{2ex}$\cEqX {{\mid}_{\cFunTyBX {R} {R} {\cB}}}
      {\cFunAbsX {x} {N_{\cSetTy {R}}} {\cFunAbsX {y} {N_{\cSetTy {R
          }}}
        {\cForsomeX {z} {N_{\cSetTy {R}}} {\cEqX {\cBinX {x} {*} {z}}
             {y}}}}}$
   \hfill (divides).

  \item[] \textsf{Thm22-via-PA-to-COF}: $\cForallX {x} {N_{\cSetTy
      {R}}} {\cBinX {x} {{\mid}_{\cFunTyBX {R} {R} {\cB}}} {0}}$
    \hfill ($0$ is top).
```

```
\ei
}
{\textsf{COF-dev-6}.}
{\textsf{PA-to-COF-1}.}
\end{group-transport}
```

Group Transportation 3.12 (Transport of | and "0 is Top" to COF)

Name: Divides-and-0-is-top-via-PA-to-COF.

Source development: PA-dev-4.

Target development: COF-dev-5.

Development morphism: PA-to-COF.

Definitions and theorems:

Def6: $\mid_{N \rightarrow N \rightarrow o} = \lambda\, x : N \,.\, \lambda\, y : N \,.\, \exists\, z : N \,.\, x * z = y$ (divides).

Thm22: $\forall\, x : N \,.\, x \mid_{N \rightarrow N \rightarrow o} 0$ (0 is top).

Transported definitions and theorems:

Def6-via-PA-to-COF:
$\mid_{R \rightarrow R \rightarrow o} = \lambda\, x : N_{\{R\}} \,.\, \lambda\, y : N_{\{R\}} \,.\, \exists\, z : N_{\{R\}} \,.\, x * z = y$ (divides).

Thm22-via-PA-to-COF: $\forall\, x : N_{\{R\}} \,.\, x \mid_{R \rightarrow R \rightarrow o} 0$ (0 is top).

New target development: COF-dev-6.

New development morphism: PA-to-COF-1.

4 LaTeX Source File

```
%% LaTeX Macros for Alonzo Notation
%%
%% William M. Farmer
%%
%% McMaster University
%%
%% April 24, 2024

%% Requires:
```

```
\usepackage{amssymb}
\usepackage{amsmath}
\usepackage{phonetic}
\usepackage{xcolor}

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% MISCELLANEOUS MACROS

\newcommand{\mName}[1]{\mathsf{#1}}
\newcommand{\mSet}[1]{\{ #1 \}}
\newcommand{\mTuple}[1]{( #1 )}
\newcommand{\mList}[1]{[ #1 ]}
\newcommand{\mSeq}[1]{\langle #1 \rangle}
\newcommand{\mSeqlike}[1]{\mSeq{\!\mSeq{#1}\!}}
\newcommand{\mAbs}[1]{\lvert #1 \rvert}
\newcommand{\mNorm}[1]{\lVert #1 \rVert}
\newcommand{\mDot}{\mathrel{.}}

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% FORMAL NOTATION

% Types

\newcommand{\fBoolTy}{\mName{BoolTy}}
\newcommand{\fBaseTy}[1]{\mName{BaseTy}(#1)}
\newcommand{\fFunTy}[2]{\mName{FunTy}(#1,#2)}
\newcommand{\fProdTy}[2]{\mName{ProdTy}(#1,#2)}

% Expressions

\newcommand{\fVar}[2]{\mName{Var}(#1,#2)}
\newcommand{\fCon}[2]{\mName{Con}(#1,#2)}
\newcommand{\fEq}[2]{\mName{Eq}(#1,#2)}
\newcommand{\fFunApp}[2]{\mName{FunApp}(#1,#2)}
\newcommand{\fFunAbs}[3]{\mName{FunAbs}(\fVar{#1}{#2},#3)}
\newcommand{\fDefDes}[3]{\mName{DefDes}(\fVar{#1}{#2},#3)}
\newcommand{\fOrdPair}[2]{\mName{OrdPair}(#1,#2)}

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% COMPACT NOTATION

% Types
```

```
\newcommand{\cBoolTy}{\omicron}
\newcommand{\cB}{\cBoolTy}
\newcommand{\cBaseTy}[1]{#1}
\newcommand{\cFunTy}[2]{({#1} \rightarrow {#2})}
\newcommand{\cFunTyX}[2]{{#1} \rightarrow {#2}}
\newcommand{\cFunTyB}[3]{\cFunTy {#1} {\cFunTyX {#2} {#3}}}
\newcommand{\cFunTyBX}[3]{\cFunTyX {#1} {\cFunTyX {#2} {#3}}}
\newcommand{\cFunTyC}[4]{\cFunTy {#1} {\cFunTyX {#2} {\cFunTyX {#3}
    {#4}}}}
\newcommand{\cFunTyCX}[4]{\cFunTyX {#1} {\cFunTyX {#2} {\cFunTyX
    {#3} {#4}}}}
\newcommand{\cProdTy}[2]{({#1} \times {#2})}
\newcommand{\cProdTyX}[2]{{#1} \times {#2}}
\newcommand{\cProdTyB}[3]{\cProdTy {#1} {\cProdTyX {#2} {#3}}}
\newcommand{\cProdTyBX}[3]{\cProdTyX {#1} {\cProdTyX {#2} {#3}}}
\newcommand{\cProdTyC}[4]{\cProdTy {#1} {\cProdTyX {#2} {\cProdTyX
    {#3} {#4}}}}
\newcommand{\cProdTyCX}[4]{\cProdTyX {#1} {\cProdTyX {#2} {\cProdTyX
     {#3} {#4}}}}

% Expressions

\newcommand{\cVar}[2]{({#1} : {#2})}
\newcommand{\cVarY}[2]{#1}
\newcommand{\cCon}[2]{{#1}_{#2}}
\newcommand{\cConY}[2]{#1}
\newcommand{\cEq}[2]{({#1} = {#2})}
\newcommand{\cEqX}[2]{{#1} = {#2}}
\newcommand{\cFunApp}[2]{(#1\,#2)}
\newcommand{\cFunAppX}[2]{#1\,#2}
\newcommand{\cFunAppB}[3]{(\cFunAppX {\cFunAppX {#1} {#2}} {#3})}
\newcommand{\cFunAppBX}[3]{\cFunAppX {\cFunAppX {#1} {#2}} {#3}}
\newcommand{\cFunAppC}[4]{(\cFunAppX {\cFunAppX {\cFunAppX
    {#1}{#2}}{#3}}{#4})}
\newcommand{\cFunAppCX}[4]{\cFunAppX {\cFunAppX {\cFunAppX
    {#1}{#2}}{#3}}{#4}}
\newcommand{\cFunAbs}[3]{(\lambda\, #1 : #2 \mDot #3)}
\newcommand{\cFunAbsX}[3]{\lambda\, #1 : #2 \mDot #3}
\newcommand{\cDefDes}[3]{(\mathrm{I}\, #1 : #2 \mDot #3)}
\newcommand{\cDefDesX}[3]{\mathrm{I}\, #1 : #2 \mDot #3}
\newcommand{\cOrdPair}[2]{(#1,#2)}

% Boolean Operators
```

```
\newcommand{\cTPC}{T_\cB}
\newcommand{\cT}{\cTPC}
\newcommand{\cFPC}{F_\cB}
\newcommand{\cF}{\cFPC}
\newcommand{\cAndPC}{\wedge_{\cFunTyBX{\cB}{\cB}{\cB}}}
\newcommand{\cAnd}[2]{({#1} \wedge {#2})}
\newcommand{\cAndX}[2]{{#1} \wedge {#2}}
\newcommand{\cAndB}[3]{\cAnd {#1} {\cAndX {#2} {#3}}}
\newcommand{\cAndBX}[3]{\cAndX {#1} {\cAndX {#2} {#3}}}
\newcommand{\cAndL}[1]{(#1)} % separator is $\And$
\newcommand{\cAndLX}[1]{#1} % separator is $\And$
\newcommand{\cImpliesPC}{\Rightarrow_{\cFunTyBX {\cB} {\cB} {\cB}}}
\newcommand{\cImplies}[2]{({#1} \Rightarrow {#2})}
\newcommand{\cImpliesX}[2]{{#1} \Rightarrow {#2}}
\newcommand{\cNegPC}{\neg_{\cFunTyX{\cB}{\cB}}}
\newcommand{\cNeg}[1]{(\neg{#1})}
\newcommand{\cNegX}[1]{\neg{#1}}
\newcommand{\cOrPC}{\vee_{\cFunTyBX{\cB}{\cB}{\cB}}}
\newcommand{\cOr}[2]{({#1} \vee {#2})}
\newcommand{\cOrX}[2]{{#1} \vee {#2}}
\newcommand{\cOrB}[3]{\cOr {#1} {\cOrX {#2} {#3}}}
\newcommand{\cOrBX}[3]{\cOrX {#1} {\cOrX {#2} {#3}}}
\newcommand{\cOrL}[1]{(#1)} % separator is $\Or$
\newcommand{\cOrLX}[1]{#1}  % separator is $\Or$

% Binary Operators

\newcommand{\cBin}[3]{({#1} \mathrel{#2} {#3})}
\newcommand{\cBinX}[3]{{#1} \mathrel{#2} {#3}}
\newcommand{\cBinB}[5]{({#1} \mathrel{#2} {#3} \mathrel{#4} {#5})}
\newcommand{\cBinBX}[5]{{#1} \mathrel{#2} {#3} \mathrel{#4} {#5}}
\newcommand{\cIff}[2]{({#1} \Leftrightarrow {#2})}
\newcommand{\cIffX}[2]{{#1} \Leftrightarrow {#2}}
\newcommand{\cNotEq}[2]{({#1} \not= {#2})}
\newcommand{\cNotEqX}[2]{{#1} \not= {#2}}

% Quantifiers

\newcommand{\cForall}[3]{(\forall\, #1 : #2 \mDot #3)}
\newcommand{\cForallX}[3]{\forall\, #1 : #2 \mDot #3}
\newcommand{\cForallB}[5]{(\forall\, #1 : #2,\, #3 : #4 \mDot #5)}
\newcommand{\cForallBX}[5]{\forall\, #1 : #2,\, #3 : #4 \mDot #5}
\newcommand{\cForallC}[7]{(\forall\, #1 : #2,\, #3 : #4,\, #5 : #6 \
    mDot #7)}
\newcommand{\cForallCX}[7]{\forall\, #1 : #2,\, #3 : #4,\, #5 : #6 \
```

```
    mDot #7}
\newcommand{\cForsome}[3]{(\exists\, #1 : #2 \mDot #3)}
\newcommand{\cForsomeX}[3]{\exists\, #1 : #2 \mDot #3}
\newcommand{\cForsomeB}[5]{(\exists\, #1 : #2,\, #3 : #4 \mDot #5)}
\newcommand{\cForsomeBX}[5]{\exists\, #1 : #2,\, #3 : #4 \mDot #5}
\newcommand{\cForsomeC}[7]{(\exists\, #1 : #2,\, #3 : #4,\, #5 : #6
    \mDot #7)}
\newcommand{\cForsomeCX}[7]{\exists\, #1 : #2,\, #3 : #4,\, #5 : #6
    \mDot #7}
\newcommand{\cForsomeUnique}[3]{(\exists!\, #1 : #2 \mDot #3)}
\newcommand{\cForsomeUniqueX}[3]{\exists!\, #1 : #2 \mDot #3}

% Definedness

\newcommand{\cBotPC}[1]{\bot_{#1}}
\newcommand{\cEmpFunPC}[2]{\Delta_{\cFunTyX {#1} {#2}}}
\newcommand{\cIsDef}[1]{(#1{\downarrow})}
\newcommand{\cIsDefX}[1]{#1{\downarrow}}
\newcommand{\cIsUndef}[1]{(#1{\uparrow})}
\newcommand{\cIsUndefX}[1]{#1{\uparrow}}
\newcommand{\cQuasiEq}[2]{({#1} \simeq {#2})}
\newcommand{\cQuasiEqX}[2]{{#1} \simeq {#2}}
\newcommand{\cNotQuasiEq}[2]{({#1} \not\simeq {#2})}
\newcommand{\cNotQuasiEqX}[2]{{#1} \not\simeq {#2}}
\newcommand{\cIfThenElse}[3]{\mName{IF}(#1,#2,#3)}
\newcommand{\cIf}[3]{(#1 \mapsto #2 \mid #3)}
\newcommand{\cIfX}[3]{#1 \mapsto #2 \mid #3}

% Sets

\newcommand{\cSetTy}[1]{\mSet{#1}}
\newcommand{\cIn}[2]{({#1} \in {#2})}
\newcommand{\cInX}[2]{{#1} \in {#2}}
\newcommand{\cNotIn}[2]{({#1} \not\in {#2})}
\newcommand{\cNotInX}[2]{{#1} \not\in {#2}}
\newcommand{\cSet}[3]{\mSet{{{#1} : {#2}} \mid {#3}}}
\newcommand{\cEmpSetPC}[1]{\emptyset_{\cSetTy {#1}}}
\newcommand{\cEmpSetAltPC}[1]{\mSet{\,}_{\cSetTy {#1}}}
\newcommand{\cUnivSetPC}[1]{U_{\cSetTy {#1}}}
\newcommand{\cFinSet}[2]{\textsf{{$#1$}-{$#2$}-SET}}
\newcommand{\cFinSetL}[1]{\mSet{#1}} % separator is ","
\newcommand{\cSubseteqPC}[1]{\subseteq_{\cFunTyBX {\cSetTy #1} {\
    cSetTy #1} {\cB}}}
\newcommand{\cSubseteq}[2]{\cBin {#1} {\subseteq} {#2}}
\newcommand{\cSubseteqX}[2]{\cBinX {#1} {\subseteq} {#2}}
```

```
\newcommand{\cUnionPC}[1]{\cup_{\cFunTyBX {\cSetTy #1} {\cSetTy #1}
    {\cSetTy #1}}}
\newcommand{\cUnion}[2]{\cBin {#1} {\cup} {#2}}
\newcommand{\cUnionX}[2]{\cBinX {#1} {\cup} {#2}}
\newcommand{\cIntersPC}[1]{\cap_{\cFunTyBX {\cSetTy #1} {\cSetTy #1}
     {\cSetTy #1}}}
\newcommand{\cInters}[2]{\cBin {#1} {\cap} {#2}}
\newcommand{\cIntersX}[2]{\cBinX {#1} {\cap} {#2}}
\newcommand{\cComplPC}[1]{\overline{\,\cdot\,}_{\cFunTyX {\cSetTy
    #1} {\cSetTy #1}}}
\newcommand{\cCompl}[1]{\big(\,\overline{#1}\,\big)}
\newcommand{\cComplX}[1]{\overline{#1}}
\newcommand{\cSetDiffPC}[1]{\setminus_{\cFunTyBX {\cSetTy #1} {\
    cSetTy #1} {\cSetTy #1}}}
\newcommand{\cSetDiff}[2]{\cBin {#1} {\setminus} {#2}}
\newcommand{\cSetDiffX}[2]{\cBinX {#1} {\setminus} {#2}}

% Tuples

\newcommand{\cTupleTyL}[1]{(#1)} % separator is $\times$
\newcommand{\cTupleL}[1]{(#1)} % separator is ","
\newcommand{\cFstPC}[2]{\mName{fst}_{\cFunTyX {\cProdTy {#1} {#2}}
    {#1}}}
\newcommand{\cSndPC}[2]{\mName{snd}_{\cFunTyX {\cProdTy {#1} {#2}}
    {#2}}}

% Functions

\newcommand{\cDomPC}[2]{\mName{dom}_{\cFunTyX {\cFunTy {#1} {#2}} {\
    cSetTy {#1}}}}
\newcommand{\cRanPC}[2]{\mName{ran}_{\cFunTyX {\cFunTy {#1} {#2}} {\
    cSetTy {#2}}}}
\newcommand{\cSubfuneqPC}[2]{\sqsubseteq_{\cFunTyBX {\cFunTy {#1}
    {#2}} {\cFunTy {#1} {#2}} {\cB}}}
\newcommand{\cFunCompPC}[3]{\circ_{\cFunTyBX {\cFunTy {#1} {#2}} {\
    cFunTy {#2} {#3}} {\cFunTy {#1} {#3}}}}
\newcommand{\cFunComp}[2]{({#1} \circ {#2})}
\newcommand{\cFunCompX}[2]{#1 \circ {#2}}
\newcommand{\cRestrictPC}[2]{|_{\cFunTyBX {\cFunTy {#1} {#2}} {\
    cSetTy {#1}} {\cFunTy {#1} {#2}}}}
\newcommand{\cRestrict}[2]{(#1 |_{#2})}
\newcommand{\cRestrictX}[2]{#1 |_{#2}}

% Miscellaneous Notation
```

```
\newcommand{\cTotal}[1]{\mName{TOTAL}(#1)}
\newcommand{\cTotalB}[1]{\mName{TOTAL2}(#1)}
\newcommand{\cSurj}[1]{\mName{SURJ}(#1)}
\newcommand{\cSurjB}[1]{\mName{SURJ2}(#1)}
\newcommand{\cInj}[1]{\mName{INJ}(#1)}
\newcommand{\cInjB}[1]{\mName{INJ2}(#1)}
\newcommand{\cBij}[1]{\mName{BIJ}(#1)}
\newcommand{\cDistinctL}[1]{\mName{DISTINCT}(#1)} % separator is ","

% Quasitypes

\newcommand{\cFunAbsQTy}[3]{\cFunAbs {#1} {#2} {#3}}
\newcommand{\cFunAbsQTyX}[3]{\cFunAbsX {#1} {#2} {#3}}
\newcommand{\cForallQTy}[3]{\cForall {#1} {#2} {#3}}
\newcommand{\cForallQTyX}[3]{\cForallX {#1} {#2} {#3}}
\newcommand{\cForallQTyB}[5]{\cForallB {#1} {#2} {#3} {#4} {#5}}
\newcommand{\cForallQTyBX}[5]{\cForallBX {#1} {#2} {#3} {#4} {#5}}
\newcommand{\cForsomeQTy}[3]{\cForsome {#1} {#2} {#3}}
\newcommand{\cForsomeQTyX}[3]{\cForsomeX {#1} {#2} {#3}}
\newcommand{\cForsomeQTyB}[5]{\cForsomeB {#1} {#2} {#3} {#4} {#5}}
\newcommand{\cForsomeQTyBX}[5]{\cForsomeBX {#1} {#2} {#3} {#4} {#5}}
\newcommand{\cDefDesQTy}[3]{\cDefDes {#1} {#2} {#3}}
\newcommand{\cDefDesQTyX}[3]{\cDefDesX {#1} {#2} {#3}}
\newcommand{\cIsDefInQTy}[2]{({#1} \downarrow {#2})}
\newcommand{\cIsDefInQTyX}[2]{{#1} \downarrow {#2}}
\newcommand{\cIsUndefInQTy}[2]{({#1} \uparrow {#2})}
\newcommand{\cIsUndefInQTyX}[2]{{#1} \uparrow {#2}}
\newcommand{\cFunQTyPC}[2]{\rightarrow_{\cFunTyBX {\cSetTy {#1}} {\
    cSetTy {#2}} {\cSetTy {\cFunTyX {#1} {#2}}}}}
\newcommand{\cFunQTy}[2]{\cFunTy {#1} {#2}}
\newcommand{\cFunQTyX}[2]{\cFunTyX {#1} {#2}}
\newcommand{\cFunQTyB}[3]{\cFunTyB {#1} {#2} {#3}}
\newcommand{\cFunQTyBX}[3]{\cFunTyBX {#1} {#2} {#3}}
\newcommand{\cFunQTyC}[4]{\cFunTyC {#1} {#2} {#3} {#4}}
\newcommand{\cFunQTyCX}[4]{\cFunTyCX {#1} {#2} {#3} {#4}}
\newcommand{\cProdQTyPC}[2]{\times_{\cFunTyBX {\cSetTy {#1}} {\
    cSetTy {#2}} {\cSetTy {\cProdTyX {#1} {#2}}}}}
\newcommand{\cProdQTy}[2]{\cProdTy {#1} {#2}}
\newcommand{\cProdQTyX}[2]{\cProdTyX {#1} {#2}}
\newcommand{\cProdQTyB}[3]{\cProdTyB {#1} {#2} {#3}}
\newcommand{\cProdQTyBX}[3]{\cProdTyBX {#1} {#2} {#3}}
\newcommand{\cProdQTyC}[4]{\cProdTyC {#1} {#2} {#3} {#4}}
\newcommand{\cProdQTyCX}[4]{\cProdTyCX {#1} {#2} {#3} {#4}}
\newcommand{\cSetQTy}[1]{{\cal P}(#1)}
\newcommand{\cTotalOn}[3]{\textsf{TOTAL-ON}(#1,#2,#3)}
```

```
\newcommand{\cTotalOnB}[4]{\textsf{TOTAL-ON2}(#1,#2,#3,#4)}
\newcommand{\cSurjOn}[3]{\textsf{SURJ-ON}(#1,#2,#3)}
\newcommand{\cSurjOnB}[4]{\textsf{SURJ-ON2}(#1,#2,#3,#4)}
\newcommand{\cInjOn}[2]{\textsf{INJ-ON}(#1,#2)}
\newcommand{\cInjOnB}[3]{\textsf{INJ-ON2}(#1,#2,#3)}
\newcommand{\cBijOn}[3]{\textsf{BIJ-ON}(#1,#2,#3)}
\newcommand{\cInf}[1]{\mName{INF}(#1)}
\newcommand{\cFin}[1]{\mName{FIN}(#1)}
\newcommand{\cCount}[1]{\mName{COUNT}(#1)}

% Dependent Quasitypes

\newcommand{\cPiTy}[2]{\cFunTyBX {\cSetTy {#1}} {\cFunTy {#1} {\
    cSetTy {#2}}} {\cSetTy {\cFunTyX {#1} {#2}}}}
\newcommand{\cPiPC}[2]{\Pi_{\cPiTy {#1} {#2}}}
\newcommand{\cPi}[3]{(\Pi\, #1 : #2 \mDot #3)}
\newcommand{\cPiX}[3]{\Pi\, #1 : #2 \mDot #3}
\newcommand{\cSigmaTy}[2]{\cFunTyBX {\cSetTy {#1}} {\cFunTy {#1} {\
    cSetTy {#2}}} {\cSetTy {\cProdTyX {#1} {#2}}}}
\newcommand{\cSigmaPC}[2]{\Sigma_{\cSigmaTy {#1} {#2}}}
\newcommand{\cSigma}[3]{(\Sigma\, #1 : #2 \mDot #3)}
\newcommand{\cSigmaX}[3]{\Sigma\, #1 : #2 \mDot #3}

% Sequences

\newcommand{\cSequencesPC}[2]{\mName{sequences}_{\cSetTy {\cFunTyX
    {#1} {#2}}}}
\newcommand{\cSeqQTy}[1]{\mSeqlike{#1}}
\newcommand{\cStreamsPC}[2]{\mName{streams}_{\cSetTy {\cFunTyX {#1}
    {#2}}}}
\newcommand{\cSeqInfQTy}[1]{\mSeq{#1}}
\newcommand{\cListsPC}[2]{\mName{lists}_{\cSetTy {\cFunTyX {#1}
    {#2}}}}
\newcommand{\cSeqFinQTy}[1]{\mList{#1}}
\newcommand{\cConsPC}[2]{\mName{cons}_{\cFunTyBX {#2} {\cFunTy {#1}
    {#2}} {\cFunTy {#1} {#2}}}}
\newcommand{\cCons}[2]{({#1} :: {#2})}
\newcommand{\cConsX}[2]{{#1} :: {#2}}
\newcommand{\cHdPC}[2]{\mName{hd}_{\cFunTyX {\cFunTy {#1} {#2}}
    {#2}}}
\newcommand{\cTlPC}[2]{\mName{tl}_{\cFunTyX {\cFunTy {#1} {#2}} {\
    cFunTy {#1} {#2}}}}
\newcommand{\cNilPC}[2]{\mName{nil}_{\cFunTyX {#1} {#2}}}
\newcommand{\cEmpListPC}[2]{{\mList{\;}}_{\cFunTyX {#1} {#2}}}
\newcommand{\cListL}[1]{\mList{#1}} % separator is ","
```

```
\newcommand{\cLenPC}[2]{\mName{len}_{\cFunTyX {\cFunTy {#1} {#2}}
    {#1}}}
\newcommand{\cLen}[1]{\mAbs {#1}}
\newcommand{\cAppendPC}[2]{\mName{++}_{\cFunTyBX {\cFunTy {#1} {#2}}
     {\cFunTy {#1} {#2}} {\cFunTy {#1} {#2}}}}
\newcommand{\cAppend}[2]{({#1} \mathrel{++} {#2})}
\newcommand{\cAppendX}[2]{{#1} \mathrel{++} {#2}}
\newcommand{\cNlistsPC}[2]{\mName{nlists}_{\cFunTyX {#1} {\cSetTy {\
    cFunTyX {#1} {#2}}}}}
\newcommand{\cSeqNFinQTy}[2]{\mList{#1}^{#2}}

% Real Numbers

\newcommand{\cRecip}[1]{\big( {#1}^{-1} \big)}
\newcommand{\cRecipX}[1]{{#1}^{-1}}
\newcommand{\cFrac}[2]{\big( \frac {#1} {#2} \big)}
\newcommand{\cFracX}[2]{\frac {#1} {#2}}
\newcommand{\cAbs}[1]{\mAbs {#1}}
\newcommand{\cSqrt}[1]{\sqrt {#1}}
\newcommand{\cNorm}[1]{\mNorm {#1}}
\newcommand{\cSum}[4]{\Big( \sum\limits_{{#1} = {#2}}^{#3} {#4} \Big
    )}
\newcommand{\cSumX}[4]{\sum\limits_{{#1} = {#2}}^{#3} {#4}}
\newcommand{\cProd}[4]{\Big( \prod\limits_{{#1} = {#2}}^{#3} {#4} \
    Big)}
\newcommand{\cProdX}[4]{\prod\limits_{{#1} = {#2}}^{#3} {#4}}
\newcommand{\cLim}[3]{\Big( \lim\limits_{{#1} \to {#2}} {#3} \Big)}
\newcommand{\cLimX}[3]{\lim\limits_{{#1} \to {#2}} {#3}}
\newcommand{\cRightLim}[3]{\Big( \lim\limits_{{#1} \to {#2}^+} {#3}
    \Big)}
\newcommand{\cRightLimX}[3]{\lim\limits_{{#1} \to {#2}^+} {#3}}
\newcommand{\cLeftLim}[3]{\Big( \lim\limits_{{#1} \to {#2}^-} {#3} \
    Big)}
\newcommand{\cLeftLimX}[3]{\lim\limits_{{#1} \to {#2}^-} {#3}}
\newcommand{\cLimSeq}[2]{\Big( \lim\limits_{{#1} \to \infty} {#2} \
    Big)}
\newcommand{\cLimSeqX}[2]{\lim\limits_{{#1} \to \infty} {#2}}
\newcommand{\cIntegral}[4]{\Big( \int_{#1}^{#2} {#3}\,d{#4} \Big)}
\newcommand{\cIntegralX}[4]{\int_{#1}^{#2} {#3}\,d{#4}}

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% THEOREM ENVIRONMENTS

\newtheorem{thm}{Theorem}[section]
```

```
\newtheorem{cor}[thm]{Corollary}
\newtheorem{lem}[thm]{Lemma}
\newtheorem{prop}[thm]{Proposition}
\newtheorem{eg}[thm]{Example}
\newtheorem{rem}[thm]{Remark}

\newtheorem{thydef}[thm]{Theory Definition}
\newtheorem{thyext}[thm]{Theory Extension}
\newtheorem{indtypethyext}[thm]{Inductive Type Theory Extension}
\newtheorem{devdef}[thm]{Development Definition}
\newtheorem{devext}[thm]{Development Extension}
\newtheorem{thytransdef}[thm]{Theory Translation Definition}
\newtheorem{thytransext}[thm]{Theory Translation Extension}
\newtheorem{devtransdef}[thm]{Development Translation Definition}
\newtheorem{devtransext}[thm]{Development Translation Extension}
\newtheorem{deftransport}[thm]{Definition Transportation}
\newtheorem{thmtransport}[thm]{Theorem Transportation}
\newtheorem{grouptransport}[thm]{Group Transportation}

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% ENVIRONMENTS

\newenvironment{theory-def}[5]
{
\color{brown!90!black}
\begin{thydef}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[]\hspace{-3ex}\textbf{Name:} #2
\item[]\hspace{-3ex}\textbf{Base types:} #3
\item[]\hspace{-3ex}\textbf{Constants:} #4
\item[]\hspace{-3ex}\textbf{Axioms:}
\end{itemize}
 #5
\end{thydef}
}

\newenvironment{theory-ext}[6]
{
\color{brown!90!black}
\begin{thyext}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[]\hspace{-3ex}\textbf{Name:} #2
```

```
\item[]\hspace{-3ex}\textbf{Extends\ } #3
\item[]\hspace{-3ex}\textbf{New base types:} #4
\item[]\hspace{-3ex}\textbf{New constants:} #5
\item[]\hspace{-3ex}\textbf{New axioms:}
\end{itemize}
#6
\end{thyext}
}

\newenvironment{ind-type-theory-ext}[5]
{
\color{brown!90!black}
\begin{indtypethyext}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[]\hspace{-3ex}\textbf{Name:} #2
\item[]\hspace{-3ex}\textbf{Extends\ } #3
\item[]\hspace{-3ex}\textbf{New base type:} #4
\item[]\hspace{-3ex}\textbf{Constructors:}
\end{itemize}
#5
\end{indtypethyext}
}

\newenvironment{dev-def}[4]
{
\color{brown!90!black}
\begin{devdef}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[]\hspace{-3ex}\textbf{Name:} #2
\item[]\hspace{-3ex}\textbf{Bottom theory:} #3
\item[]\hspace{-3ex}\textbf{Definitions and theorems:}
\end{itemize}
#4
\end{devdef}
}

\newenvironment{dev-ext}[4]
{
\color{brown!90!black}
\begin{devext}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[]\hspace{-3ex}\textbf{Name:} #2
```

```
\item[]\hspace{-3ex}\textbf{Extends\ } #3
\item[]\hspace{-3ex}\textbf{New definitions and theorems:}
\end{itemize}
#4
\end{devext}
}

\newenvironment{theory-trans-def}[6]
{
\color{brown!90!black}
\begin{thytransdef}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[]\hspace{-3ex}\textbf{Name:} #2
\item[]\hspace{-3ex}\textbf{Source theory:} #3
\item[]\hspace{-3ex}\textbf{Target theory:} #4
\item[]\hspace{-3ex}\textbf{Base type mapping:}
\end{itemize}
#5
\begin{itemize}
\item[]\hspace{-3ex}\textbf{Constant mapping:}
\end{itemize}
#6
\end{thytransdef}
}

\newenvironment{theory-trans-ext}[7]
{
\color{brown!90!black}
\begin{thytransext}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[]\hspace{-3ex}\textbf{Name:} #2
\item[]\hspace{-3ex}\textbf{Extends\ } #3
\item[]\hspace{-3ex}\textbf{New source theory:} #4
\item[]\hspace{-3ex}\textbf{New target theory:} #5
\item[]\hspace{-3ex}\textbf{New base type mapping:}
\end{itemize}
#6
\begin{itemize}
\item[]\hspace{-3ex}\textbf{New constant mapping:}
\end{itemize}
#7
\end{thytransext}
}
```

```
\newenvironment{dev-trans-def}[6]
{
\color{brown!90!black}
\begin{devtransdef}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[]\hspace{-3ex}\textbf{Name:} #2
\item[]\hspace{-3ex}\textbf{Source development:} #3
\item[]\hspace{-3ex}\textbf{Target development:} #4
\item[]\hspace{-3ex}\textbf{Base type mapping:}
\end{itemize}
#5
\begin{itemize}
\item[]\hspace{-3ex}\textbf{Constant mapping:}
\end{itemize}
#6
\end{devtransdef}
}

\newenvironment{dev-trans-ext}[6]
{
\color{brown!90!black}
\begin{devtransext}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[]\hspace{-3ex}\textbf{Name:} #2
\item[]\hspace{-3ex}\textbf{Extends\ } #3
\item[]\hspace{-3ex}\textbf{New source development:} #4
\item[]\hspace{-3ex}\textbf{New target development:} #5
\item[]\hspace{-3ex}\textbf{New defined constant mapping:}
\end{itemize}
#6
\end{devtransext}
}

\newenvironment{def-transport}[9]
{
\color{brown!90!black}
\begin{deftransport}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[]\hspace{-3ex}\textbf{Name:} #2
\item[]\hspace{-3ex}\textbf{Source development:} #3
\item[]\hspace{-3ex}\textbf{Target development:} #4
```

```
\item[]\hspace{-3ex}\textbf{Development morphism:} #5
\item[]\hspace{-3ex}\textbf{Definition:}
\end{itemize}
#6
\begin{itemize}
\item[]\hspace{-3ex}\textbf{Transported definition:}
\end{itemize}
#7
\begin{itemize} \setlength{\itemsep}{0pt}
\item[]\hspace{-3ex}\textbf{New target development:} #8
\item[]\hspace{-3ex}\textbf{New development morphism:} #9
\end{itemize}
\end{deftransport}
}

\newenvironment{thm-transport}[8]
{
\color{brown!90!black}
\begin{thmtransport}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[]\hspace{-3ex}\textbf{Name:} #2
\item[]\hspace{-3ex}\textbf{Source development:} #3
\item[]\hspace{-3ex}\textbf{Target development:} #4
\item[]\hspace{-3ex}\textbf{Development morphism:} #5
\item[]\hspace{-3ex}\textbf{Theorem:}
\end{itemize}
#6
\begin{itemize}
\item[]\hspace{-3ex}\textbf{Transported theorem:}
\end{itemize}
#7
\begin{itemize}
\item[]\hspace{-3ex}\textbf{New target development:} #8
\end{itemize}
\end{thmtransport}
}

\newenvironment{group-transport}[9]
{
\color{brown!90!black}
\begin{grouptransport}[#1]\em
\noindent
\begin{itemize} \setlength{\itemsep}{0pt}
\item[]\hspace{-3ex}\textbf{Name:} #2
```

```
\item[]\hspace{-3ex}\textbf{Source development:} #3
\item[]\hspace{-3ex}\textbf{Target development:} #4
\item[]\hspace{-3ex}\textbf{Development morphism:} #5
\item[]\hspace{-3ex}\textbf{Definitions and theorems:}
\end{itemize}
#6
\begin{itemize}
\item[]\hspace{-3ex}\textbf{Transported definitions and theorems:}
\end{itemize}
#7
\begin{itemize} \setlength{\itemsep}{0pt}
\item[]\hspace{-3ex}\textbf{New target development:} #8
\item[]\hspace{-3ex}\textbf{New development morphism:} #9
\end{itemize}
\end{grouptransport}
}
```

References

[1] W. M. Farmer. *Simple Type Theory: A Practical Logic for Expressing and Reasoning About Mathematical Ideas.* Computer Science Foundations and Applied Logic. Birkhäuser, 2023.